The Cycle of the Seasons 4: Trig Equations

Part 4: Finding the Equation for Sinusoidal Data

We have found that such cyclic phenomena as average temperature and daylight hours approximate the shape of a sine curve. In fact, the graph can be approximated by a function of the form

$$f(x) = A \sin [B (x - C)] + D$$

Our next challenge will be to determine values for the constants *A*, *B*, *C* and *D.*

Amplitude (*A*)

First, we will look at *A*, the amplitude. We can find the amplitude of a sine function by taking half of the numerical difference between the highest point and the lowest point on the curve. Since we did not record the data for every day of the year, we need to refer to the U. S. Naval Observatory web site to find these values. Now compute the Amplitude for Hartford's equation, as follows:

$$A = 1/2\ (\text{max - min})$$

Record your calculation for Hartford's daylight amplitude.

Period

Next, we will try to determine *B*, which provides information on the period of the function. We have seen in class that the fundamental sine function repeats its pattern every 2π units; that is, it has a period of 2π. What is the period of the Coventry daylight function? (After how many days does the length-of-day pattern repeat itself?) You should see that this is a value much greater than 2π, so we will need to stretch out the standard sine wave if we want to match the Coventry daylight pattern. This "stretching factor" is the value of the variable *B* in our equation, and it can be calculated by making a ratio of the standard period (2π) to the period we want, as follows:

$B = 2\pi/x$, where x is the period of our Coventry daylight function

How are you doing? Why not take a graphing calculator and graph what you have so far:

$$\boldsymbol{f(x) = A \sin Bx}$$

using the values of *A* and *B* that you determined, and compare this graph with the daylight graph you completed in Part 4. You should see a curve that has the correct size and shape, but that is in the wrong place. If your size and shape are correct, continue on to the next challenge: determing the value of *D*. (We will skip *C*, for the moment.)

Vertical Shift (*D*)

The variable *D*, gives the 'average' value of the sine wave. When you graphed your equation above, you probably found that your graph was too low, oscillating about the *x*-axis instead of up where the data points are. How much higher should it be? How did you decide? Use this as the value of *D* to give your graph the vertical 'boost' it needs!

Try out the new modification on your graphing utility. Are you getting closer to the daylight function?

Phase Shift (C)

Once you determine *D*, you have the horizontal axis of the sine wave and you can work on finding *C*, which gives the *phase shift* of the function. The phase shift represents the horizontal shift of the graph compared to the fundamental sine curve.

Start by overlaying the line $y = D$ onto your graphing calculator screen (that is, enter the equation $y = D$ for the value of *D* you just calculated, above. Now determine visually how much of a horizontal shift (how many days) your graph needs to move (and in which direction) to make it match the starting point of a standard sine function. *Important note*: to write a horizontal shift to the *right,* into a mathematical formula, you subtract the value (in this case *C*,the number of days) from *x*.

Try out the completed model, $f(x) = A \sin [B(x - C)] + D$, using your values for *A*, *B*, *C*, and *D*.

This graph should fit the data points very closely. Check your 'fit', by comparing values of your graph/equation, to the actual data. Adjust the values of your variables, until you think you have the best possible fit. (An exact fit is impossible.)

Your mathematical model might be slightly different from those of others working on this lab. Remember that the data are approximations and so were the points you plotted. If you used a graphing utility to plot the points, you should notice that few of the points landed exactly where you tried to place them because there are a limited number of pixels on the screen and the graphing utility does the best it can. Much of what you did in choosing the constants involves intelligent estimation. You found a function to model the data fairly well. There is more than one such function, and no function will be a perfect match.

One difficulty inherent in finding a model is that the given pattern is not a perfect sine wave, since the apparent path of the sun is actually one that is slightly elliptical rather than perfectly circular. That's a problem we are not going to attempt to fix in this lab!
wave

Part 6b: New Tools for Comparing Time and Temperature

We have found that such cyclic phenomena as average temperature and daylight hours can be approximated by a function of the form

$$f(x) = A \sin [\, B (x - C)] + D$$

To complete your assignment:

1. Give the equation for *both* the Daylight data *and* the Average Temperature data. Explain how you determined each of the variables: *A*, *B*, *C* and *D*. Be sure to answer all the questions asked in Part 6a.
2. Determine the maximum value of each function. On what date(s) do these maximums occur?
3. Describe your findings and suggest any possible reason(s) for this phenomenon.

This POW will be graded as follows:

- A maximum grade of B, if you choose to write Part 6 as seperate addendum to your Final Report.
- A maximum grade of A+, if you choose to re-write your Final Report in light of your new findings.
- Up to five points will be deducted for each incorrect variable (*A*, *B*, *C* and *D*); up to five points will be
deducted for each incorrect maximum, and up to ten points will be deducted

for failing to explain
how you determined the variables and describe/explain your findings (#3, above).