The Cycles of Life

Many phenomena in biology appear in cycles. Often these cycles are driven by the natural physical cycles that result from the daily cycle of light or the annual cycle of the seasons. Oscillations are most easily studied using trigonometric functions. This section begins with a discussion of annual temperature variations, then we review some trigonometric functions. The emphasis for this section is modeling oscillatory behavior with the sine and cosine functions.

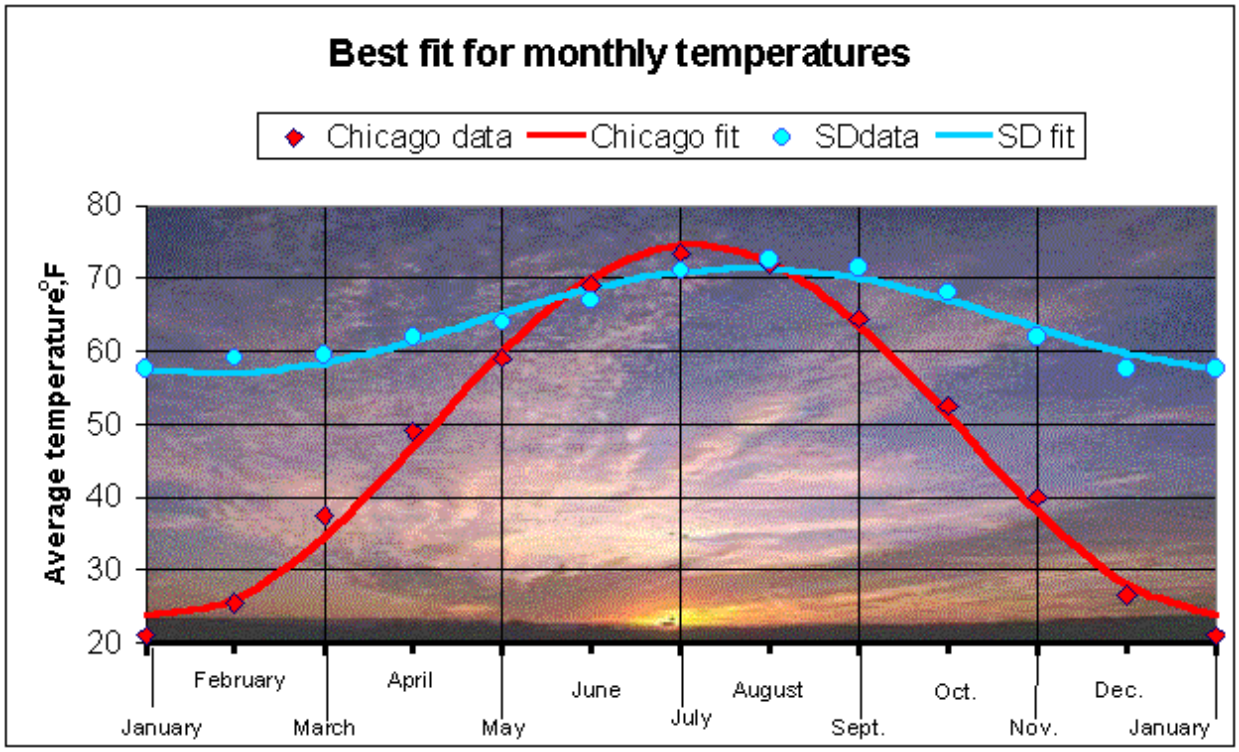
Annual Temperature Cycles

Often the weather report states what the average expected temperature of a given day is. These averages are derived from long term collection of data on weather for a particular location. Clearly, there is a relatively wide variation from these averages, but they provide approximations to the expected weather for a particular time of year. The long term averages also provide a baseline to help researchers determine the effects of global warming over the background noise of annual variation. Obviously, there are seasonal differences in the average daily temperature with higher averages in the summer and lower averages in the winter. Below we provide a table of showing the monthly average high and low temperatures for San Diego and Chicago.

Month	San Diego	Chicago
January	66/49	29/13
February	67/51	34/17
March	66/53	46/29
April	68/56	59/39
May	69/59	70/48
June	72/62	80/58
July	76/66	84/63
August	78/68	82/62
September	77/66	75/54
October	75/61	63/42
November	70/54	48/32
December	66/49	34/19

What mathematical tools can help predict the annual temperature cycles? Polynomials and exponentials do not exhibit the periodic behavior that we see for these average monthly temperatures, so these functions are not appropriate for modeling this system. The most natural

candidates for studying monthly temperatures are the trigonometric functions. Below are graphs of the average monthly temperatures for San Diego and Chicago, which are computed from the table by averaging the average high and low temperatures.



The two graphs above have some similarities and clear differences. They both show the same seasonal period as expected; however, the seasonal variation or amplitude of oscillation for Chicago is much greater than San Diego. Also, the overall average temperature for San Diego, being further south and near the ocean, is greater than the average for Chicago. The overlying models in the graph above use cosine functions. The fit using the cosine function provides a reasonable approximation though clearly there are errors due to other complicating factors in weather prediction. Before providing more details of the models for these temperature cycles, we review some basic facts about trigonometric functions.