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Activity 5.4: Curved Lines

In this activity, you expand your technological toolbox by exploring a new type of mode on your calculator. It will allow you to describe a growth process either recursively or in closed-form, and lets you control the intervals that determine the recursive description. You also investigate how each of the control numbers that are used to describe exponential growth in closed-form affects their graphs.

PART I: SEQ MODE

A **sequence** is a pattern of numbers identified by a label called the **term number**. Usually the term numbers are the counting numbers {1, 2, 3, …} or the whole numbers {0, 1, 2, 3, …}. Often the explanatory variable in a function rule is a term number and the resulting response variable values are the sequence values.

TAKE NOTE

You used sequences in Chapter 1, *Secret Codes*, where the position number in the alphabet was the term number. Most of the animations using time as the parameter were sequences. Even the yearly population growth of the moose form sequences, with the year number being the term number.

The calculator allows you to generate sequence values by working in **SEQ** mode (rather than **FUNC** mode). The steps for working with a calculator in SEQ mode are summed up in Handout H5.6. In this part of the activity you will become familiar with SEQ mode. You will explore a situation that features both additive and multiplicative growth processes at work:

> *Your sister just received $100 for her birthday and your parents want her to save it for college. To encourage her, they are willing to give her an extra $12 every year. However, all the money is to be kept in a cookie jar. The alternative is to use their bank, which will pay her 10% of the amount invested as interest. Any money earned in interest is deposited into the account automatically.*

1. Use SEQ mode on your calculator. Enter a recursive equation to describe the growth pattern in the cookie jar. Enter a closed-form equation to describe the growth pattern in the bank savings account. Record the equations that were used.
2. Use the TABLE feature of the calculator to record the balance in both savings plans for a 10-year time interval.
3. Use home-screen iteration to verify the values you recorded from the TABLE.
4. Graph both equations on the same calculator screen. Record the WINDOW setting that you used and sketch the screen display.
5. Use your work to make an investment recommendation. Which savings plan should she use for saving the money?

FYI

In SEQ mode on the TI, the variable button (X,T,θ,n) will generate the variable n for the Index number. You are restricted to calling the sequences u, v or w; any recursive reference must use 2nd –7, 2nd –8 or 2nd –9 to work with previous sequence values.

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PART II: GRAPHS OF EXPONENTIAL FUNCTIONS

The exponential equation $y = a \cdot b^x$ includes two control numbers, a and b. Use a spreadsheet or graphing calculator to carry out a detailed investigation to determine the main effects of each constant on the graph of the closed-form equation. If you use a spreadsheet, be sure to use the closed-form equation, and to include values of x other than just integers.

6. Keep the starting value, a, constant and change the growth factor, b. (Work in FUNC mode.)

   a) Choose a value for the control number a, and one for b. What is your equation?

   b) Graph your equation. Record the WINDOW used, and sketch the graph.

   c) Choose several other values for b. Display their graphs in the same WINDOW. Sketch a picture of the graphs that are produced and record the equations used to create each graph.

7. What features of the graph are controlled by the growth factor?

8. In general, the graph is not very sensitive to changes in the value of b over a short time interval. However, two specific values are particularly interesting because small changes have great effects. To look for these values, graph two exponential equations having bases that are *almost* the same. If the two graphs are practically the same, look somewhere else. If the graphs are very *different*, you have found that the value is in between the two bases that were used.

   a) Find these two values; describe what happens to the graph as you change b past each of these numbers.

   b) Explain from the context of population growth why these effects are reasonable.

9. Now, keep the growth factor, b, constant and change the starting value, a.

   a) To have a basis of comparison for the previous work, start with the same values for a and b that were used in Question 6(a). What is your equation again?

   b) Graph your equation. Record the WINDOW used, and sketch the graph.

   c) Choose several other values for a. Display their graphs in the same WINDOW. Sketch a picture of the graphs that are produced and record the equations used to create each graph.

10. What is the primary feature controlled by a?

11. There is one particular value of a near which the function is sensitive. Find that value and describe what happens as a moves past it.

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Activity Summary

In this activity, you:

- learned how to use SEQ mode on your calculator and solved a problem involving two investment strategies.
- explored how the graphs of closed-form exponential equations are affected by the control numbers a and b.
- found that a determines where the graph intersects the y-axis, whereas b affects the general shape of the curve, as well as whether it approaches the x-axis for positive or negative values of x.

DISCUSSION/REFLECTION:

1. What is the difference between the graph of the sequence $A_n = 20(1.20)^n$ and that of the function $y = 20(1.20)^x$?
2. When should you use a sequence, as opposed to a function?
3. Refer back to the work done on the two investment strategies in Part I. Explain how to tell when an additive model has the same value as a multiplicative model.

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