Individual Work 5.3: One Pattern—Many Ways to Describe It

This set of exercises will give you opportunities to develop and use recursive equations for multiplicative growth processes. It will also provide you a chance to develop and use exponential equations.

1. Write each of the following descriptions as a recursive process.

   a) The first number in a pattern is 10; each successive number is 1.3 times as big.

   b) The first number in a pattern is 30; each successive term is 20% bigger.

   c) The closed-form equation describing a pattern of numbers is $A = 200(1.05)^t$.

2. Each of the following descriptions can be considered a closed-form process.

   a) Rewrite $(1.28) \cdot (1.28) \cdot (1.28) \cdot (45) \cdot (1.28) \cdot (1.28)$ using exponential notation.

   b) The starting number (A_0) in a pattern is 35; each successive number is found by multiplying the previous one by 1.75. Write an equation in closed-form to describe this pattern.

   c) Rewrite the recursive-form equation given by $A_{n+1} = A_n \cdot 1.10$, $A_0 = 58$, as a closed-form equation.

3. Anita deposits $250 in a savings account. The account manager informs her that it will grow at the rate of 4.5% per year (from earning interest).

   a) What is the relative rate and growth factor (per year) for this situation?

   b) Use the growth factor to find the account balance after one year. Did the balance go up by 4.5% during that year? Explain.

   c) Predict what the percent of increase would be over a two-year period. Explain. Then find the account balance at the end of the second year. Did the balance go up by 4.5% during the second year?

   d) What was the actual rate of increase over the two-year period? (How could you have gotten that answer more directly?)

4. Continue working with Anita's bank account from Question 3.

a) Use home-screen iteration to generate the account balance for the first five years. Record those values in a table.

b) What recursive equation describes the growth of the account balance?

c) Make a graph of the current balance versus the previous balance. How can you tell from this graph that the growth in the account balance is multiplicative?

d) Make a graph of the change in the account balance versus the previous balance. How can you tell from this graph that the growth is multiplicative?

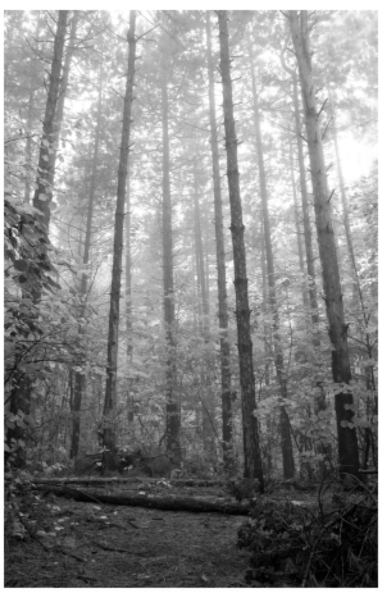
5. Continue working with Anita's bank account from Question 3.

a) What closed-form equation describes the same growth?

b) If no additional money was added or deposited, what would the account balance be after 10, 15, 20, and 25 years?

c) Using 5-year intervals make a graph of the account balance versus the number of years that have elapsed. How is the shape of the graph determined by the fact that this growth is multiplicative?

d) If Anita had deposited only \$125, instead of \$250, in the account, what would the account balance be after 25 years? How did you get that answer?



6. Suppose that a state buys a large forest to set aside as an animal refuge. Information on past populations does not exist for any animals. So, the state wildlife manager begins keeping detailed records. At the time of purchase, the population of one species is around 250. One year later, it is around 280. Assume that the growth of this species is a multiplicative process described by the recursive model: $P_{current} = P_{previous} + r \cdot P_{previous}$.

a) Determine the growth factor and the relative rate for this situation.

b) Draw an arrow diagram for this recursive process. Then use the arrow diagram to calculate the population of the species after another year has elapsed.

c) Use the arrow diagram to predict what the population of the species *would* have been, one year before the forest was purchased.

d) Write a closed-form equation to describe the population growth. To get the answer for (c), what would you use as the value for the variable representing the number of years? Verify your answer by evaluating your closed-form equation with that value.

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7. Dante thinks that a pattern of growth going up by 10% each time can be projected *backwards* by having it go down by 10% each time.

   a) Start with a "nice" number, 100. Increase it by 10%, and then decrease that answer by 10%. Do you get back to 100? Explain.

   b) Maybe the order makes a difference. Start with 100 again, only this time decrease it by 10%, and then increase that answer by 10%. Do you get back to 100? Explain.

   c) What is it about this situation that prevents Dante's idea from working?

   d) If you want to go backwards through a pattern of growth that was made by having each term go up by 10% from the previous one, what should you do? Why?

8. Suppose $P(0) = 20.7$, $P(1) = 26.9$, $P(2) = 35$, $P(3) = 45.5$ and $P(5) = 76.9$.

   a) What kind of growth is the quantity represented by P undergoing? Explain.

   b) What is the value of $P(4)$? How did you get that answer?

The break from working on a reproduction model to describe the moose population in Adirondack State Park has proven useful. Like most investigations, it raises some questions as it answers others. For example, is the pattern of growth that was found in Activity 5.3 due to the assumption of reproduction or the choice of numbers used? That can be settled by exploring a similar scenario with slightly different numbers.

9. Start with an initial population of 27 moose. Assume that 2/3 of the population are females, that each female has a baby every year, and 2/3 of the babies born each year are female. (The ratios were selected as a convenience, and probably do not reflect the actual moose growth.)

   a) Make a data table to track the population growth over the next two years.

   b) Is the population growth a multiplicative process? Explain.

10. Write equations to describe the situation in Question 9, using the specified quantities.

   a) P and n

   b) $P_{current}$, $P_{previous}$, P_0 and r

   c) $P_{current}$, $P_{previous}$, P_0 and b

11. Make graphs to describe the situation in Question 9, using the specified quantities.

a) P versus n

b) ΔP versus $P_{current}$

c) P_{next} versus $P_{current}$

12. Use the closed-form equation, your answer to Question 10(a), to predict the population after 3 years. Then extend the data table used in Question 9 one additional row, and fill in all the entries. What happens to the model?

Activity 5.3 introduced three distinct ways of describing a multiplicative process. That may seem confusing, but it gives you options for describing situations involving this type of growth. Just as with linear forms, you need to know *what goes where*. Here is an opportunity to practice that.

13. Given the closed-form exponential equation $A = 500(1.1)^t$, where A is the amount of money you have in the bank after t years.

    a) What is the growth factor and relative rate of increase per year?

    b) How much money did you put into the bank when the account was opened (at time $t = 0$)?

    c) How much money would you have after 5 years (at time $t = 5$)?

    d) Write a recursive equation to describe the same situation.

14. Given a typical recursive equation:
    $A_{current} = A_{previous} + 0.25 \cdot A_{previous}$, with $A_0 = 200$.

    a) What is the growth factor and relative rate of increase per time period?

    b) If this were written as a closed-form equation, what would be the base?

    c) Write a recursive equation to describe this same situation with a single calculation.

    d) After one time period, what would be the amount? Explain how to arrive at that answer in two different ways.

15. A bank account is growing exponentially with an initial deposit of $600 growing to a value of $630 after 1 year.

    a) What is the growth factor for this situation?

    b) What is the percent of increase for the account (relative rate of growth)?

    c) If no additional money is put into the account, but it continues to grow at the same rate, how much would be in the bank at the end of 10 years? Explain.

The closed-form equation for multiplicative processes involves the use of exponents. This is a good time to explore problems that illustrate the **laws of exponents** here.

One of the laws of exponents provides a short-cut for multiplying two terms that have the same base raised to powers. If the problem is $(b^n) \cdot (b^m)$, think of the first term as being $(1 \cdot b^n)$. You start with an initial value of 1, and multiply it by some other number, b, a total of n times, to get some new value. That answer can be thought of as the new initial value to be multiplied m more times by the same growth factor, b. Altogether, you would be multiplying by that growth factor a total of $(n + m)$ times. The algebraic rule that says the same thing is:

$$(b^n) \cdot (b^m) = b^{n+m}$$

16. Suppose Tania deposits $1 in an account that pays 5% interest on the balance at the end of each year. The plan is to keep the money in the account for a certain length of time. Later she decides to keep the money in the account for a longer period of time.

    a) Assume the first time interval is 5 years, followed by another 2 years. What is the mathematical relationship between the growth factor for the first 5 years, the one for the next 2 years, and the one for the overall time period of 7 years?

    b) Now assume the first time interval is 4 years, followed by another 3 years. What is the relationship between the individual growth factors and the overall one (for all 7 years) in this instance?

    c) What do you notice about the overall growth factor (over all 7 years) in each of the two specific cases? Does that make sense in this problem?

    d) Rewrite the relationships that you found in parts (a) and (b) using exponents.

17. There are other laws of exponents to consider.

    a) Explain how to simplify $(b^n)/(b^m)$ using the same kind of reasoning as in the discussion before Question 16.

    b) For positive values of b, what does b^0 have to equal?

    c) Combine your rule from part (a) with your answer to part (b) to give a new rule for simplifying $(b^0)/(b^n)$. What does this new rule say about negative exponents?

    d) Devise a similar rule that explains how to simplify $(b^n)^m$.

18. Use the Law of Exponents to find the value for each of the following exponential expressions. Check your answers by rewriting the problem without exponents inside of parentheses and finding the value of the new expression obtained.

a) $(2^5) \cdot (2^2)$

b) $(3^6)/(3^2)$

c) $(2^3)^4$

19. Use the laws of exponents to simplify each of the following variable expressions:

a) $(x^6) \cdot (x^3) \cdot (x^2)$

b) $\frac{a^{10}}{a^4}$

c) $(t^3)^5$

20. Dottie wrote down an answer of 2.8561 after raising a number to a power. She forgot what the base was, but recalled that the exponent was a 4.

a) Write the problem as an exponential equation. In terms of the order of operations, what does the equation say to do?

b) How can you undo the process of raising a number to a power? In other words, how can you solve the equation you wrote as your answer to part (a)?

c) What base did she use to get the answer of 2.8561? Verify your solution to the equation by raising it to the fourth power.