PROBLEM A6.1

Growing Up

Looking at the way plants grow, you can see that there are many different kinds of growth.

If you look at **Figure 1** very carefully, you can find a rule for the growth of this plant. There are two different types of branches: the ones that go up (U) and the ones that go sideways (S) to the left or to the right. Branching always and only occurs opposite a leaf.

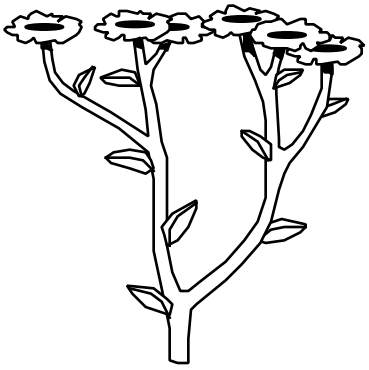

Figure 1.
The branching of a plant.

The rules for the branching are:

- If a branch is of the U-type, it produces two new branches, one of the U-type and one of the S-type.
- If a branch is of the S-type, then it produces only a new branch of the U-type.

1. Check the rule in Figure 1.

This way of growing can be drawn more schematically, as in **Figure 2**.

To the right of this plant you find a time counter, *t*. Assume this plant grows steadily: at moment $t = 0$ it starts growing, at $t = 1$ it makes its first new branch, at $t = 2$ the new branch produces new branches, and so on.

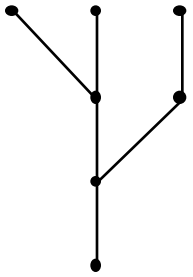

Figure 2.
An idealized drawing of a branching plant.

Thus, you can draw the growth of this plant over time and you can say something about the number of different types of branches at each moment.

2. Complete the drawing of the plant as it will look at $t = 5$.

3. Complete the table shown in **Figure 3**.

Time t	# New branches of U-type	# New branches of S-type	Total number of new branches
0	1	0	1
1	1	1	2
2	2	1	3
3			
4			
5			

Figure 3.
Growth table for an idealized plant.

4. Predict the number of S-type branches and U-type branches for $t = 6$, 7, and 8.

5. $U(t)$ represents the number of new U-type branches, $S(t)$ represents the number of new S-type branches, and $B(t)$ stands for the total number of new branches, each at time t.

   Find recursive formulas for $U(t)$, $S(t)$ and $B(t)$. In your formulas, use only $U(t-1)$ and $S(t-1)$.