6.8 Inverse Trig

Practice Tasks

Pictured: "Through the tri-oval" by Curtis Palmer
https://commons.wikimedia.org

I. Concepts and Procedures

1. Solve the following equations. Approximate values of the inverse trigonometric functions to the thousandths place, where x refers to an angle measured in radians.

   a. $5 = 6\cos(x)$

   b. $-\frac{1}{2} = 2\cos\left(x - \frac{\pi}{4}\right) + 1$

   c. $1 = \cos\big(3(x-1)\big)$

   d. $1.2 = -0.5\cos(\pi x) + 0.9$

   e. $7 = -9\cos(x) - 4$

   f. $2 = 3\sin(x)$

   g. $-1 = \sin\left(\frac{\pi(x-1)}{4}\right) - 1$

   h. $\pi = 3\sin(5x+2) + 2$

   i. $\frac{1}{9} = \frac{\sin(x)}{4}$

   j. $\cos(x) = \sin(x)$

   k. $\sin^{-1}(\cos(x)) = \frac{\pi}{3}$

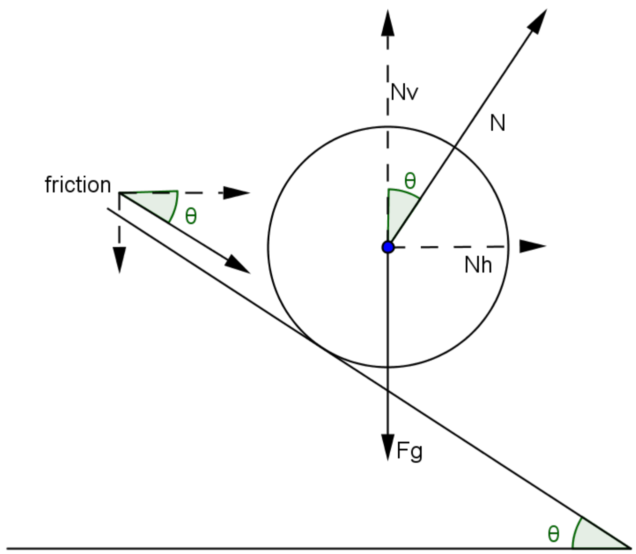
   l. $\tan(x) = 3$

   m. $-1 = 2\tan(5x+2) - 3$

   n. $5 = -1.5\tan(-x) - 3$

II. Problem Solving

1. Talladega Superspeedway has some of the steepest turns in all of NASCAR. The main turns have a radius of about 305 m and are pitched at 33°. Let N be the perpendicular force on the car, and N_v and N_h be the vertical and horizontal components of this force, respectively. See the diagram below.



   a. Let μ represent the coefficient of friction; recall that μN gives the force due to friction. To maintain the position of a vehicle traveling around the bank, the centripetal force must equal the horizontal force in the direction of the center of the track. Add the horizontal component of friction to the horizontal component of the perpendicular force on the car to find the centripetal force. Set your expression equal to $\frac{mv^2}{r}$, the centripetal force.

   b. Let μ represent the coefficient of friction; recall that μN gives the force due to friction. To maintain the position of a vehicle traveling around the bank, the centripetal force must equal the horizontal force in the direction of the center of the track. Add the horizontal component of friction to the horizontal component of the perpendicular force on the car to find the centripetal force. Set your expression equal to $\frac{mv^2}{r}$, the centripetal force.

c. Add the vertical component of friction to the force due to gravity, and set this equal to the vertical component of the perpendicular force.

d. Solve one of your equations in part (a) or part (b) for m, and use this with the other equation to solve for v.

e. Assume $\mu = 0.75$, the standard coefficient of friction for rubber on asphalt. For the Talladega Superspeedway, what is the maximum velocity on the main turns? Is this about how fast you might expect NASCAR stock cars to travel? Explain why you think NASCAR takes steps to limit the maximum speeds of the stock cars.

f. Does the friction component allow the cars to travel faster on the curve or force them to drive slower? What is the maximum velocity if the friction coefficient is zero on the Talladega roadway?

g. Do cars need to travel slower on a flat roadway making a turn than on a banked roadway? What is the maximum velocity of a car traveling on a 305 m turn with no bank?

2. Let the velocity v in miles per second of a particle in a particle accelerator after t seconds be modeled by the function $v = \tan\left(\frac{\pi t}{6000} - \frac{\pi}{2}\right)$ on an unknown domain.

   a. What is the t-value of the first vertical asymptote to the right of the y-axis?

   b. If the particle accelerates to 99% of the speed of light before stopping, then what is the domain?
   Note: $c \approx 186000$. Round your solution to the ten-thousandths place.

   c. How close does the domain get to the vertical asymptote of the function?

   d. How long does it take for the particle to reach the velocity of Earth around the sun (about 18.5 miles per second)?

   e. What does it imply that v is negative up until $t = 3000$?

3. Consider the situation of sitting down with eye level at 46 in.. Find the missing distances and heights for the following:
   a. The bottom of the picture is at 50 in. and the top is at 74 in. What is the optimal viewing distance?

   b. The bottom of the picture is at 52 in. and the top is at 60 in. What is the optimal viewing distance?

   c. The bottom of the picture is at 48 in. and the top is at 64 in. What is the optimal viewing distance?

   d. What is the height of the picture if the optimal viewing distance is 1 ft. and the bottom of the picture is hung at 47 in.?

4. Consider the situation where you are looking at a painting a inches above your line of sight and b inches below your line of sight.

   a. Find the optimal viewing distance if it exists.

   b. If the average standing eye height of Americans is 61.4 in., at what height should paintings and other works of art be hung?

5. The amount of daylight per day is periodic with respect to the day of the year. The function
$y = -3.016\cos\left(\frac{2\pi x}{365}\right) + 12.25$ gives the number of hours of daylight in New York, y, as a function of the number of days since the winter solstice (December 22), which is represented by x.

   a. On what days will the following hours of sunlight occur?
      i. 15 hours, 15 minutes.
      ii. 12 hours.
      iii. 9 hours, 15 minutes.
      iv. 10 hours.
      v. 9 hours.

   b. Give a function that will give the day of the year from the solstice as a function of the hours of daylight.

   c. What is the domain of the function you gave in part (b)?

   d. What does the domain tell you in the context of the problem?

   e. What is the range of the function? Does this make sense in the context of the problem? Explain.

III. Reasoning

1. In general, since the cosine function is merely the sine function under a phase shift, mathematicians and scientists regularly choose to use the sine function to model periodic phenomena instead of a mixture of the two. What behavior in data would prompt a scientist to use a tangent function instead of a sine function?

IV. Modeling

1. Ocean tides are an example of periodic behavior. At a particular harbor, data was collected over the course of 24 hours to create the following model: $y = 1.236\sin\left(\frac{\pi}{3}x\right) + 1.798$, which gives the water level, y, in feet above the MLLW (mean lower low water) as a function of the time, x, in hours.

    a. How many periods are there each day?

    b. Write a function that gives the time in hours as a function of the water level. How many other times per day will have the same water levels as those given by the function?

2. A particle is moving along a line at a velocity of $y = 3\sin\left(\frac{2\pi x}{5}\right) + 2\,\frac{\mathrm{m}}{\mathrm{s}}$ at location x meters from the starting point on the line for $0 \le x \le 20$.

    a. Find a formula that represents the location of the particle given its velocity.

    b. What is the domain and range of the function you found in part (a)?

    c. Use your answer to part (a) to find where the particle is when it is traveling $5\,\frac{\mathrm{m}}{\mathrm{s}}$ for the first time.

    d. How can you find the other locations the particle is traveling at this speed?

3. A vehicle with a mass of 500 kg rolls down a slanted road with an acceleration of $0.04\,\frac{\mathrm{m}}{\mathrm{s}^2}$. The frictional force between the wheels of the vehicle and the road is 1800 Newtons.
   a. Sketch the situation.

   b. What is the angle of elevation of the road?

   c. The steepness of a road is frequently measured as grade, which expresses the slope of a hill as a percent that the change in height is of the change in horizontal distance. What is the grade of the hill described in this problem?

4. Canton Avenue in Pittsburgh, PA is considered to be one of the steepest roads in the world with a grade of 37%.
   a. Assuming no friction on a particularly icy day, what would be the acceleration of a 1000 kg car with only gravity acting on it?

   b. The force due to friction is equal to the product of the force perpendicular to the road and the coefficient of friction μ. For icy roads of a non-moving vehicle, assume the coefficient of friction is $\mu = 0.3$. Find the force due to friction for the car above. If the car is in park, will it begin sliding down Canton Avenue if the road is this icy?

   c. Assume the coefficient of friction for moving cars on icy roads is $\mu = 0.2$. What is the maximum angle of road that the car will be able to stop on?

5. At a particular harbor over the course of 24 hours, the following data on peak water levels was collected (measurements are in feet above the MLLW):

Time	$1:30$	$7:30$	$14:30$	$20:30$
Water Level	-0.211	8.21	-0.619	7.518

a. What appears to be the average period of the water level?

b. What appears to be the average amplitude of the water level?

c. What appears to be the average midline for the water level?

d. Fit a curve of the form $y = A\sin\big(\omega(x-h)\big) + k$ or $y = A\cos\big(\omega(x-h)\big) + k$ modeling the water level in feet as a function of the time.

e. According to your function, how many times per day will the water level reach its maximum?

f. How can you find other time values for a particular water level after finding one value from your function?

g. Find the inverse function associated with the function in part (d). What is the domain and range of this function? What type of values does this function output?