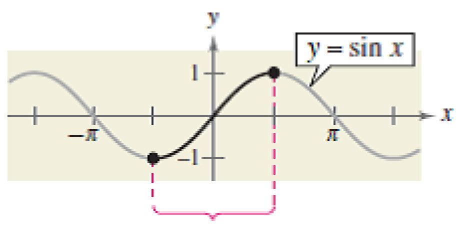
6.8: Can I Get An Inverse?

Inverse Trigonometric Functions

Pictured: "Through the tri-oval" by Curtis Palmer
https://commons.wikimedia.org

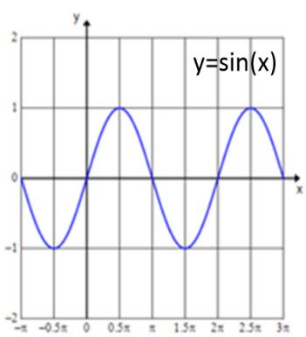
Inverse trigonometric functions can be used to model the changing angles of banked turns and flat straightaways to which racecar drivers repeatedly adjust. The Talladega Superspeedway, pictured at right, has some of the steepest turns in all of NASCAR.

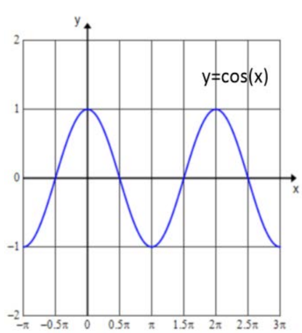
Recall from Unit 4 that the inverse of a function f is a function f^{-1} that reverses the rule of f. For a function to have an inverse, it must be one-to-one. Since the trigonometric functions are not one-to-one, they do not have inverses. It is possible, however, to restrict the domains of the trigonometric functions to make it one-to-one.

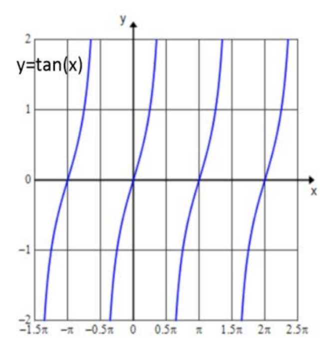


sin *x* has an inverse function on this interval

1. Use the graphs of the sine, cosine, and tangent functions to answer each of the following questions.







a. State the domain of each function.

b. Would the inverse of the sine, cosine, or tangent functions also be functions? Explain.

c. For each function, select a suitable domain that will make the function invertible.

2. Consider the function $f(x) = \sin(x), -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.
    a. State the domain and range of this function.

    b. Find the equation of the inverse function.

    c. State the domain and range of the inverse.

3. Write an equation for the inverse cosine function, and state its domain and range.

4. Write an equation for the inverse tangent function, and state its domain and range.

5. Evaluate each of the following expressions without using a calculator. Use radian measures.

a. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

c. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

d. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

e. $\sin^{-1}(1)$

f. $\sin^{-1}(-1)$

g. $\cos^{-1}(1)$

h. $\cos^{-1}(-1)$

i. $\tan^{-1}(1)$

j. $\tan^{-1}(-1)$

6. Solve each trigonometric equation such that $0 \leq x \leq 2\pi$. Round to three decimal places when necessary.
   a. $2\cos(x) - 1 = 0$

   b. $3\sin(x) + 2 = 0$

7. Solve each trigonometric equation such that $0 \leq x \leq 2\pi$. Give answers in exact form.
   a. $\sqrt{2}\cos(x) + 1 = 0$

   b. $\tan(x) - \sqrt{3} = 0$

   c. $\sin^2(x) - 1 = 0$

8. Solve each trigonometric equation such that $0 \leq x \leq 2\pi$. Round answers to three decimal places.
   a. $5\cos(x) - 3 = 0$

   b. $3\cos(x) + 5 = 0$

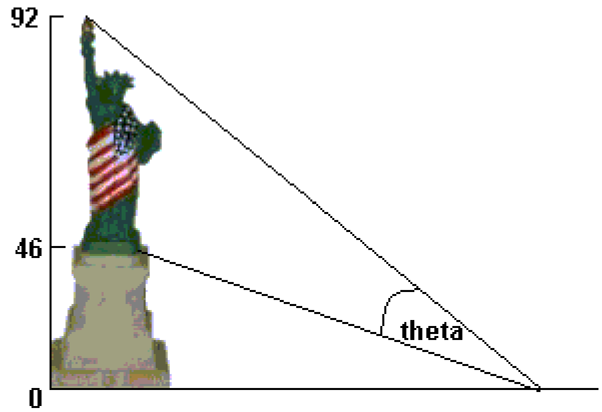
   c. $3\sin(x) - 1 = 0$

   d. $\tan(x) = -0.115$

II. Viewing Angle

9. The Statue of Liberty is 92 meters tall and sits on a pedestal that is 46 meters above the ground. An observer who is 6 feet tall wants to stand at the ideal viewing distance in front of the statue.

   a. Sketch the statue and observer. Label all appropriate measurements on the sketch, and define them in context.



   b. How far back from the statue should the observer stand so that his or her viewing angle (from the feet of the statue to the tip of the torch) is largest? What is the value of the largest viewing angle?

   c. Based on your own height, what would be your best viewing distance from the statue?

   d. If there are 66 meters of dry land in front of the statue, is the viewer still on dry land at the best viewing distance?

10. Hanging on a museum wall is a picture with base a inches above a viewer's eye level and top b inches above the viewer's eye level.

a. Model the situation with a diagram.

b. Determine an expression that could be used to find the ideal viewing distance x that maximizes the viewing angle y.

c. Find the ideal viewing distance, given the a and b values assigned to you. Calculate the maximum viewing angle in degrees.

d. Complete the table using class data, which indicates the ideal values for x given different assigned values of a and b. Note any patterns you see in the data.

a (inches)	b (inches)	x at max (inches)	y max (degrees)

III. Modeling with Inverse Trig Functions

11. A designer wants to test the safety of a wheelchair ramp she has designed for a building before constructing it, so she creates a scale model. To meet the city's safety requirements, an object that starts at a standstill from the top of the ramp and rolls down it should not experience an acceleration exceeding $2.4\frac{m}{s^2}$.

    a. A ball of mass 0.1 kg is used to represent an object that rolls down the ramp. As it is placed at the top of the ramp, the ball experiences a downward force due to gravity, which causes it to accelerate down the ramp. Knowing that the force applied to an object is the product of its mass and acceleration, create a sketch to model the ball as it accelerates down the ramp.

    b. If the ball rolls at the maximum allowable acceleration of $2.4\ \frac{\mathrm{m}}{\mathrm{s}^2}$, what is the angle of elevation for the ramp?

    c. If the designer wants to exceed the safety standards by ensuring the acceleration of the object does not exceed $2.0\frac{m}{s^2}$, by how much will the maximum angle of elevation decrease?

    d. How does the mass of the object used in the scale model affect the value of θ? Explain your response.

12. A vehicle with a mass of 1000 kg rolls down a slanted road with an acceleration of $0.07\,\frac{m}{s^2}$. The frictional force between the wheels of the vehicle and the wet concrete road is 2800 Newtons.
    a. Sketch the situation.

    b. What is the angle of elevation of the road?

    c. What is the maximum angle of elevation the road could have so that the vehicle described would not slide down the road?

13. The declination of the sun is the path the sun takes overhead the earth throughout the year. When the sun passes directly overhead, the declination is defined as 0°, while a positive declination angle represents a northward deviation and a negative declination angle represents a southward deviation. Solar declination is periodic and can be roughly estimated using the equation$\delta = -23.44°(\cos\left(\frac{360}{365}\right)(N+10))$, where N represents a calendar date, e.g., $N = 1$ is January 1, and δ is the declination angle of the sun measured in degrees.

    a. Describe the domain and range of the function.

    b. Write an equation that represents N as a function of δ.

    c. Determine the calendar date(s) for the given angles of declination:
        i. 10°
        ii. $-5.2°$
        iii. 25°

    d. When will the sun trace a direct path above the equator?

14. The average monthly temperature in a coastal city in the United States is periodic and can be modeled with the equation $y = -8\cos\left((x-1)\left(\frac{\pi}{6}\right)\right) + 17.5$, where y represents the average temperature in degrees Celsius and x represents the month, with $x = 1$ representing January.

a. Write an equation that represents x as a function of y.

b. A tourist wants to visit the city when the average temperature is closest to 25° Celsius. What recommendations would you make regarding when the tourist should travel? Justify your response.

15. The estimated size for a population of rabbits and a population of coyotes in a desert habitat are shown in the table. The estimated population sizes were recorded as part of a long-term study related to the effect of commercial development on native animal species.

Years since initial count (n)	0	3	6	9	12	15	18	21	24
Estimated number of rabbits (r)	14,989	10,055	5,002	10,033	15,002	10,204	4,999	10,002	14,985
Estimated number of coyotes (c)	1,995	2,201	2,003	1,795	1,999	2,208	2,010	1,804	2,001

a. Describe the relationship between sizes of the rabbit and coyote populations throughout the study.

b. Plot the relationship between the number of years since the initial count and the number of rabbits. Fit a curve to the data.

c. Repeat the procedure described in part (b) for the estimated number of coyotes over the course of the study.

d. During the study, how many times was the rabbit population approximately 12,000? When were these times?

e. During the study, when was the coyote population estimate below 2,100?

16. In an amusement park, there is a small Ferris wheel, called a kiddie wheel, for toddlers. The formula $H(t) = 10\sin\left(2\pi\left(t - \frac{1}{4}\right)\right) + 15$ models the height H (in feet) of the bottom-most car t minutes after the wheel begins to rotate. Once the ride starts, it lasts 4 minutes.
   a. What is the initial height of the car?
   b. How long does it take for the wheel to make one full rotation?
   c. What is the maximum height of the car?
   d. Find the time(s) on the interval $0 \leq t \leq 4$ when the car is at its maximum height.

17. Many animal populations fluctuate periodically. Suppose that a wolf population over an 8-year period is given by the function $W(t) = 800\sin\left(\frac{\pi}{4}t\right) + 2200$, where t represents the number of years since the initial population counts were made.
   a. Find the time(s) on the interval $0 \leq t \leq 8$ such that the wolf population equals 2500.
   b. On what time interval during the 8-year period is the population below 2000?
   c. Why would an animal population be an example of a periodic phenomenon?