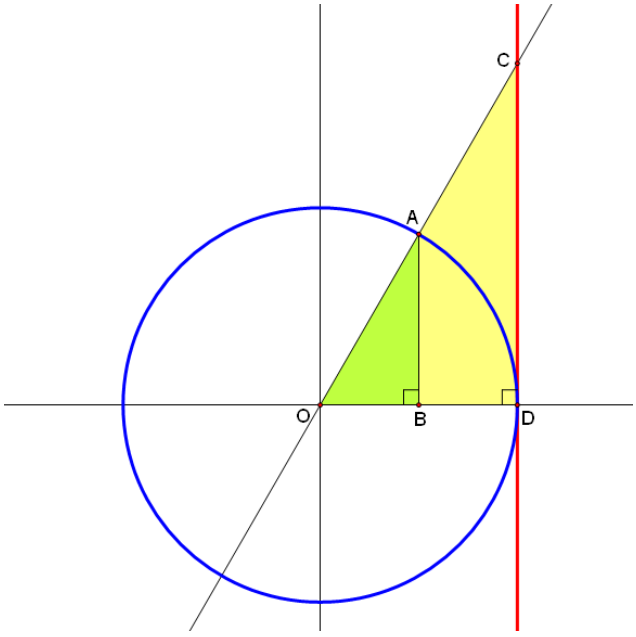
6.7: Off on a Tangent

Tangent Lines and Tangent Functions

The general form of the tangent function, which is similar to that of sinusoidal functions, is $y = a\tan(bx + c) + d$, where a produces a vertical stretch or compression, b affects the period, c produces a phase shift, d produces a vertical shift and neither a no b are 0.

In this activity you will learn how to construct a tangent line and discover properties of tangent lines and functions.

The circle shown to the right is a unit circle, and the length of $\widehat{DA}$ is $\frac{\pi}{3}$ radians.



1. $\overleftrightarrow{CD}$ is the only tangent line in the diagram. Write the geometric definition of tangent.

2. Which segment in the diagram has length $\sin\left(\frac{\pi}{3}\right)$?

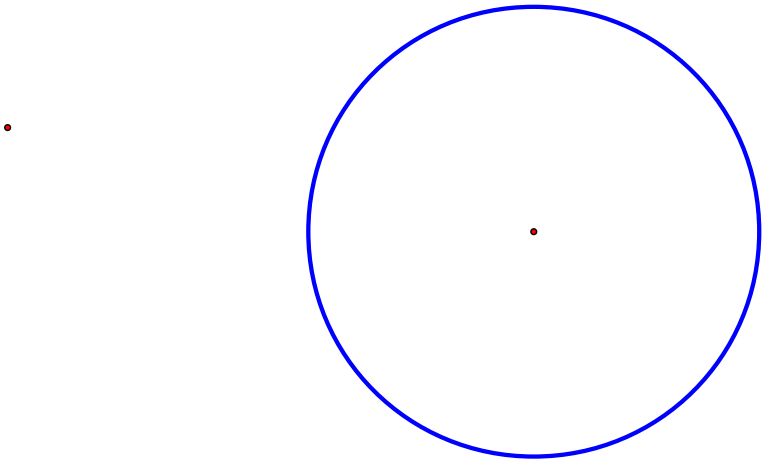
3. Which segment in the diagram has length $\cos\left(\frac{\pi}{3}\right)$?

4. Which segment in the diagram has length $\tan\left(\frac{\pi}{3}\right)$?

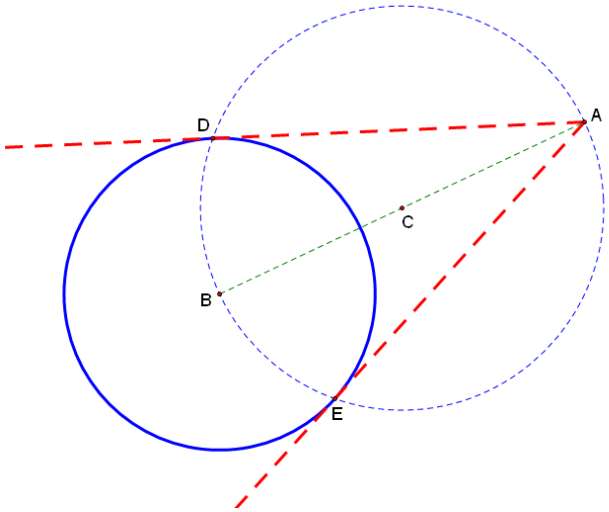
5. Which segment in the diagram has length $\sec\left(\frac{\pi}{3}\right)$?

II. Constructing a Tangent

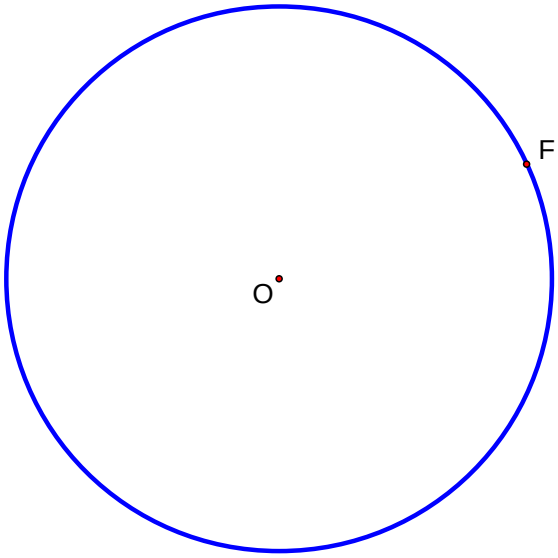
6. Use a compass to construct the tangent lines to the given circle that pass through the given point.



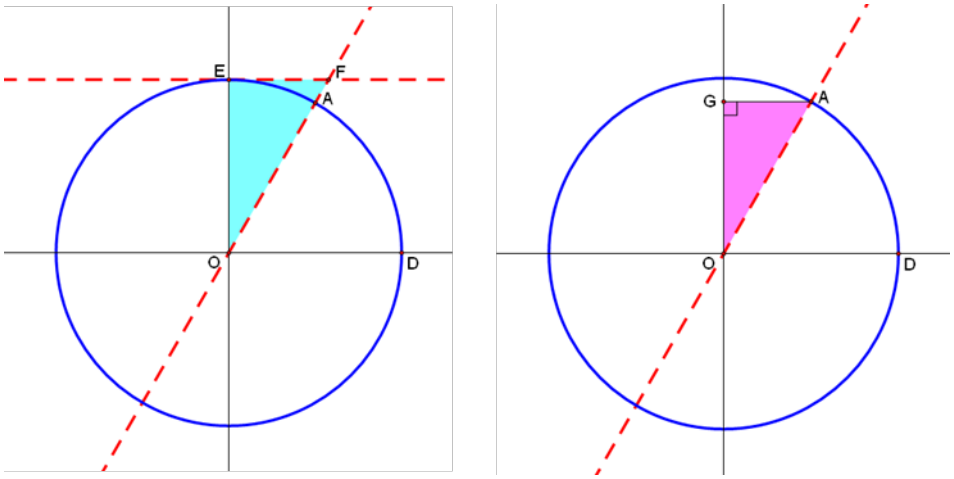
7. Analyze the construction shown below. Argue that the lines shown are tangent to the circle with center B.



8. Use a compass to construct a line that is tangent to the circle below at point F. Then choose a point G on the tangent line, and construct another tangent to the circle through G.



9. The circles shown below are unit circles, and the length of $\widehat{DA}$ is $\frac{\pi}{3}$ radians.



a. Which trigonometric function corresponds to the length of $\overline{EF}$?

b. Which trigonometric function corresponds to the length of $\overline{OF}$?

c. Which trigonometric identity gives the relationship between the lengths of the sides of ΔOEF?

d. Which trigonometric identities give the relationships between the corresponding sides of ΔOEF and ΔOGA?

10. What is the value of $\csc\left(\frac{\pi}{3}\right)$? What is the value of $\cot\left(\frac{\pi}{3}\right)$? Use the Pythagorean theorem to support your answers.