6.6 Laws of Sine and Cosine

Practice Tasks

I. Concepts and Procedures

1. Let ΔABC be the triangle with the given lengths and angle measurements. Find all possible missing measurements using the law of sines.

   $a = 5, m\angle A = 43, m\angle B = 80.$

   $a = 3.2, m\angle A = 110, m\angle B = 35.$

   $a = 9.1, m\angle A = 70, m\angle B = 95.$

   $a = 3.2, m\angle B = 30, m\angle C = 45.$

   $a = 12, m\angle B = 29, m\angle C = 31.$

2. Consider triangles with the following measurements. If two sides are given, use the law of cosines to find the measure of the third side. If three sides are given, use the law of cosines to find the measure of the angle between a and b.

   $a = 4, b = 6, C = 35.$

   $a = 2, b = 3, C = 110.$

   $a = 5, b = 5, C = 36.$

   $a = 7.5, b = 10, C = 90.$

   $a = 4.4, b = 6.2, C = 9.$

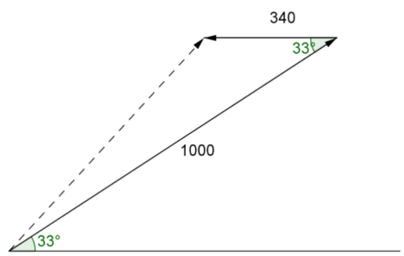
II. Problem Solving

1. A surveyor is working at a river that flows north to south. From her starting point, she sees a location across the river that is 20° north of east from her current position, she labels the position S. She moves 110 feet north and measures the angle to S from her new position, seeing that it is 32° south of east.
   a. Draw a picture representing this situation.

   b. Find the distance from her starting position to S.

   c. Explain how you can use the procedure the surveyor used in this problem (called triangulation) to calculate the distance to another object.

2. A trebuchet launches a boulder at an angle of elevation of 33° at a force of 1000 N. A strong gale wind is blowing against the boulder parallel to the ground at a force of 340 N. The figure is shown below.



   a. What is the force in the direction of the boulder's path?

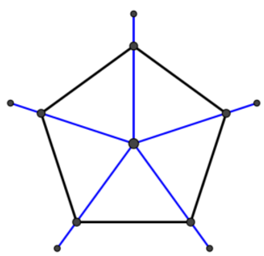
   b. What is the angle of elevation of the boulder after the wind has influenced its path?

3. Sara and Paul are on opposite sides of a building that a telephone pole fell on. The pole is leaning away from Paul at an angle of 59° and towards Sara. Sara measures the angle of elevation to the top of the telephone pole to be 22°, and Paul measures the angle of elevation to be 34°. Knowing that the telephone pole is about 35 ft. tall, answer the following questions.
   a. Draw a diagram of the situation.

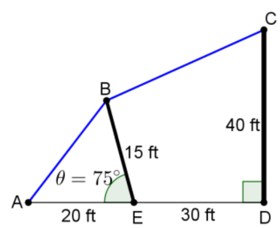
   b. How far apart are Sara and Paul?

   c. If we assume the building is still standing, how tall is the building?

4. Cliff wants to build a tent for his son's graduation party. The tent is a regular pentagon, as illustrated below. How much guide wire (show in blue) does Cliff need to purchase to build this tent? Round your answers to the nearest thousandths.

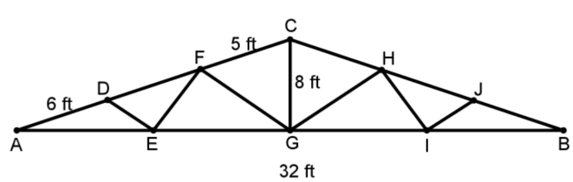


Overhead View



Side View

5. A roofing contractor needs to build roof trusses for a house. The side view of the truss is shown below. Given that G is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AG}$, I is the midpoint of $\overline{GB}$, $\overline{AB} = 32$ ft., $\overline{AD} = 6$ ft., $\overline{FC} = 5$ ft., and $\angle AGC = 90^{\circ}$. Find $\overline{DE}$, $\overline{EF}$, and $\overline{FG}$. Round your answers to the nearest thousandths.



III. Reasoning

1. Consider the case of a triangle with sides 5, 12, and the angle between them 90°.

    a. What is the easiest method to find the missing side?

    b. What is the easiest method to find the missing angles?

    c. Can you use the law of cosines to find the missing side? If so, perform the calculations. If not, show why not.

    d. Can you use the law of cosines to find the missing angles? If so, perform the calculations. If not, show why not.

    e. Consider a triangle with sides a, b, and the angle between them 90°. Use the law of cosines to prove a well-known theorem. State the theorem.

    f. Summarize what you have learned in parts (a) through (e).

2. Consider the case of two line segments $\overline{CA}$ and $\overline{CB}$ of lengths 5 and 12, respectively, with $m\angle C = 180°$.

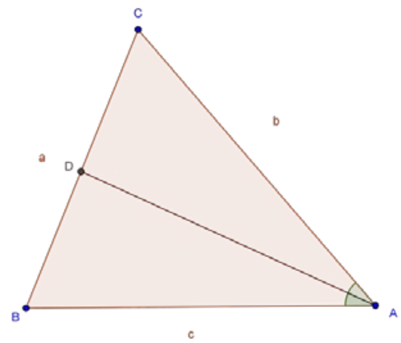
    a. Is ABC a triangle?

    b. What is the easiest method to find the distance between A and B?

    c. Can you use the law of cosines to find the distance between A and B? If so, perform the calculations. If not, show why not.

    d. Summarize what you have learned in parts (a) through (c).

3. Consider the triangle pictured below.



Use the law of sines to prove the generalized angle bisector theorem, that is, $\frac{\overline{BD}}{\overline{DC}} = \frac{c \sin(\angle BAD)}{b \sin(\angle CAD)}$. (Although this is called the generalized angle bisector theorem, we do not assume that the angle bisector of BAC intersects side $\overline{BC}$ at D. In the case that AD is an angle bisector, then the formula simplifies to $\frac{\overline{BD}}{\overline{DC}} = \frac{c}{b}$.)

a. Use the triangles ABD and ACD to express $\frac{c}{\overline{BD}}$ and $\frac{b}{\overline{DC}}$ as a ratio of sines.

b. Note that angles BDA and ADC form a linear pair. What does this tell you about the value of the sines of these angles?

c. Solve each equation in part (a) to be equal to the sine of either $\angle BDA$ or $\angle ADC$.

d. What do your answers to parts (b) and (c) tell you?

e. Prove the generalized angle bisector theorem.