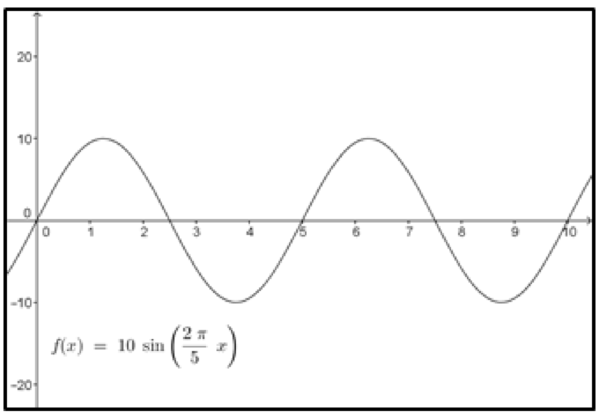
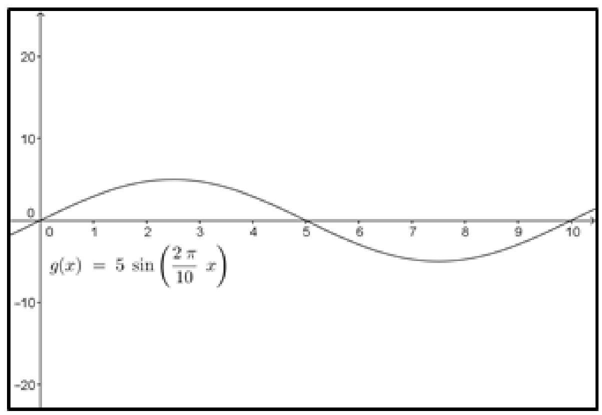
6.4 Math and Music

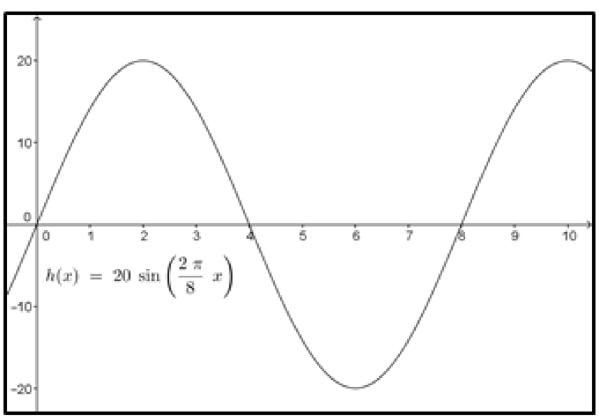
Practice Tasks

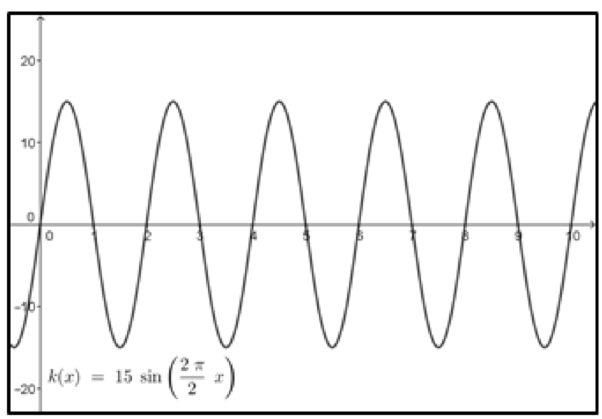
I. Concepts and Procedures

1. When you hear a musical note played on an instrument, the tones are caused by vibrations of the instrument. The vibrations can be represented graphically as a sinusoid. The **amplitude** is a measure of the loudness of the note, and the **frequency** is a measure of the pitch of the note. Recall that the frequency of a sinusoidal function is the reciprocal of its period. Louder notes have greater amplitude, and higher pitched notes have larger frequencies.
   a. State the amplitude, period, and frequency of each sinusoidal function graphed below.









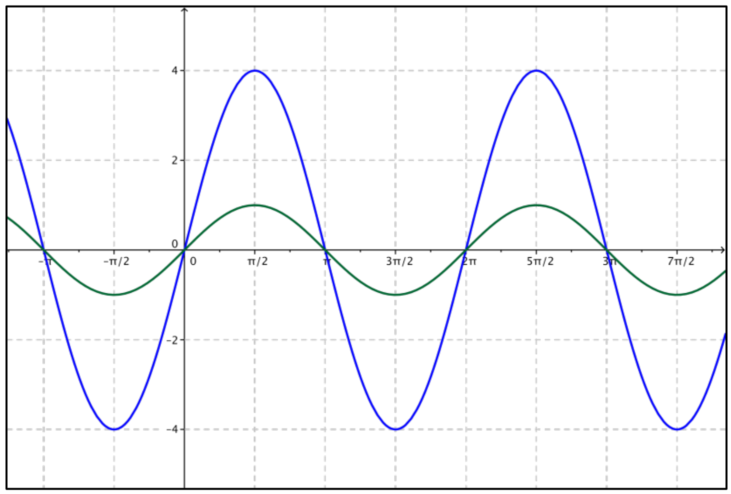
b. Order the graphs (on the previous page)from quietest note to loudest note.

c. Order the graphs from lowest pitch note to highest pitch note.

II. Problem Solving

When two musical notes are played simultaneously, wave interference occurs. Wave interference is also responsible for the actual sound of the notes that you hear.

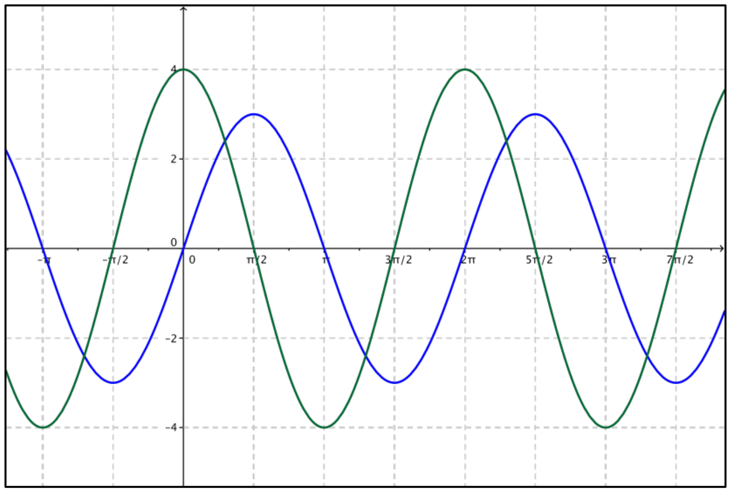
1. The graphs of two functions, f and g are shown below.



   a. Model wave interference by picking several points on the graphs of f and g and then using those points to create a graph of $h(x) = f(x) + g(x)$.

   b. What is a formula for h? Explain how you got your answer.

2. The graphs of f and g are shown below.

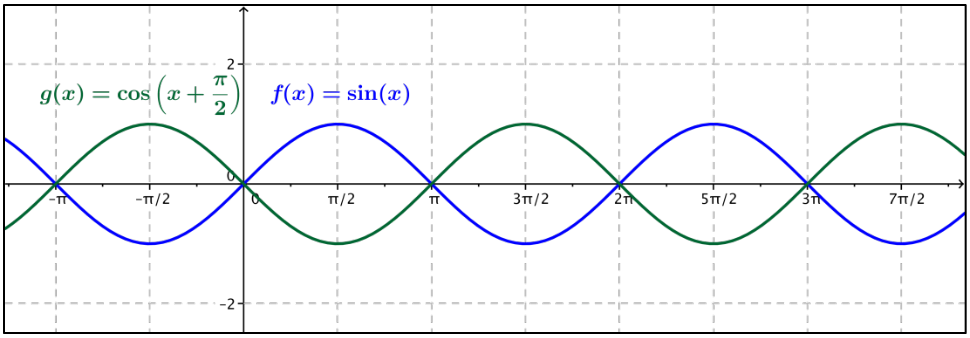


a. Model wave interference by picking several points on the graphs of f and g and then using those points to create a graph of $h(x) = f(x) + g(x)$.

b. What is an approximate formula for h? Explain how you got your answer.

3. Let $f(x) = \sin(x)$ and $g(x) = \cos\left(x + \frac{\pi}{2}\right)$.

   a. Predict what the graph of the wave interference function $h(x) = f(x) + g(x)$ would look like in this situation.



   b. Use an appropriate identity to confirm your prediction.

III. Reasoning

1. Show that in general, the function $h(x) = a\cos(bx - c)$ can be rewritten as the sum of a sine and cosine function with equal periods and different amplitudes.

2. Find an exact formula for $h(x) = 12\sin(x) + 5\cos(x)$ in the form $h(x) = a\cos(x - c)$. Graph $f(x) = 12\sin(x)$, $g(x) = 5\cos(x)$, and $h(x) = 12\sin(x) + 5\cos(x)$ together on the same axes.

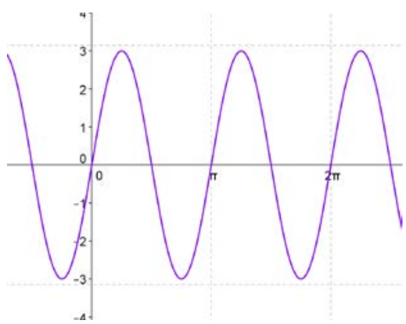
3. Find an exact formula for $h(x) = 2\sin(x) - 3\cos(x)$ in the form $h(x) = a\cos(x - c)$. Graph $f(x) = 2\sin(x)$, $g(x) = -3\cos(x)$, and $h(x) = 2\sin(x) - 3\cos(x)$ together on the same axes.

4. Can you find an exact formula for $h(x) = 2\sin(2x) + 4\sin(x)$ in the form $h(x) = a\sin(x - c)$? If not, why not? Graph $f(x) = 2\sin(2x)$, $g(x) = 4\sin(x)$, and $h(x) = 2\sin(2x) + 4\sin(x)$ together on the same axes.

IV. Modeling

1. Find a sinusoidal function $f(x) = a\sin(bx + c) + d$ that fits each of the following graphs.

a.



b.