6.2 Spaghetti Graphs

Investigation

In groups of 5-6 (determined by your teacher) complete the following investigation, and use your knowledge of the Unit Circle to discover the shape and properties of the graphs of the sine and cosine functions.

Materials:

- 1 sheet of white computer paper
- Poster Board (or 2 sheets of legal paper)
- Yarn
- Tape or Glue
- Protractor
- Compass
- Spaghetti
- Marker
- Ruler

I. Creating the Unit Circle (no spaghetti yet!)

For the first part of the activity, create a large Unit Circle, complete with the common angle measurements we have discussed in previous lessons.

1. On your white computer paper, use a ruler to draw an x and y axis. The origin (where the two axes meet) should be as close to the center of the paper as possible.

2. Use the compass to create a circle with the center at the origin and a radius of at least 3 ½ inches. Obviously, don't make your circle so large that it doesn't fit on the paper.

3. Use the protractor to measure the angles at $\frac{\pi}{6}, \frac{\pi}{4}$, and $\frac{\pi}{3}$, and make a dot where these angles are on your Unit Circle. Additionally, label the terminal point for each angle measurement. For example: the terminal point of $t = \frac{\pi}{6}$ is $(\frac{\sqrt{3}}{2}, \frac{1}{2})$.

4. Continue in a similar fashion by labeling all of the common angle measurements and their corresponding terminal points in the remaining three Quadrants. Also mark the points where the x and y axis cross the Unit Circle $(\frac{\pi}{2}, \pi,$ and $\frac{3\pi}{2})$.

II. Creating the Sine Graph

1. Wrap your piece of string around the Unit Circle. One end of the string should be placed at the point (1,0).

2. Use the marker to mark points on the string at each of the special angles.

3. Lay the two sheets of legal paper lengthwise so that they are touching (you are creating one long piece of paper). Draw a coordinate axis that spans across these two pieces of paper. Since we are graphing the function $y = \sin\theta$, label the horizontal axis θ, and the vertical axis $\sin\theta$.

4. Place the yarn on the horizontal axis with the end of the string that started at the point (1,0) now placed at the origin (this is where $\theta = 0$). On your paper, mark off the special angles ($\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, etc). Once the positive horizontal axis has been labeled, flip the string over and label the negative horizontal axis using the same marks. These represent negative angle measures ($-\frac{\pi}{6}, -\frac{\pi}{4}, -\frac{\pi}{3}$, etc).

5. Look back at your Unit Circle. Since we are interested in the sine function, which corresponds to the y value, we are interested in the distance a given point is from the horizontal axis. At the angle measure of 0, the y value is 0, so we will not need any spaghetti. However, at $\frac{\pi}{6}$, the y value is $\frac{1}{2}$. Place a noodle of spaghetti vertically with one end on the horizontal axis and the remainder of the noddle going through the point ($\frac{\sqrt{3}}{2}, \frac{1}{2}$). Break the noodle off at that point.

6. Take the piece of spaghetti and glue it onto your sine graph at $\frac{\pi}{6}$ on your horizontal axis. Since the y value is positive, the spaghetti should be placed above the horizontal axis. Repeat this process for each special angle.

7. Continue this process with the negative angles as well, keeping in mind whether or not the y value is positive or negative.

III. Creating the Cosine Graph

1. Wrap your piece of string around the Unit Circle. One end of the string should be placed at the point (1,0).
2. Use the marker to mark points on the string at each of the special angles.
3. Lay the two sheets of legal paper lengthwise so that they are touching (you are creating one long piece of paper). Draw a coordinate axis that spans across these two pieces of paper. Since we are graphing the function $x = \cos\theta$, label the horizontal axis θ, and the vertical axis $\cos\theta$.
4. Place the yarn on the horizontal axis with the end of the string that started at the point (1,0) now placed at the origin (this is where $\theta = 0$). On your paper, mark off the special angles ($\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, etc). Once the positive horizontal axis has been labeled, flip the string over and label the negative horizontal axis using the same marks. These represent negative angle measures ($-\frac{\pi}{6}, -\frac{\pi}{4}, -\frac{\pi}{3}$, etc).
5. Look back at your Unit Circle. Since we are interested in the cosine function, which corresponds to the x value, we are interested in the distance a given point is from the horizontal axis. At the angle measure of 0, the x value is 1. Place a piece of spaghetti horizontally with one end at vertical axis going through the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$. Break off the noodle at the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$.
6. Take the piece of spaghetti and glue it onto your cosine graph at 0 on your horizontal axis. Since the x value is positive, the spaghetti should be placed above the horizontal axis. Repeat this process for each special angle.
7. Continue this process with the negative angles as well, keeping in mind whether or not the x value on the Unit Circle is positive or negative.

IV. Presentation

Make a presentation to the class of your work, including a detailed mathematical explanation of how the graphs of the sine and cosine functions relate to the Unit Circle. Make predictions about how the secant and cosecant graphs should look.