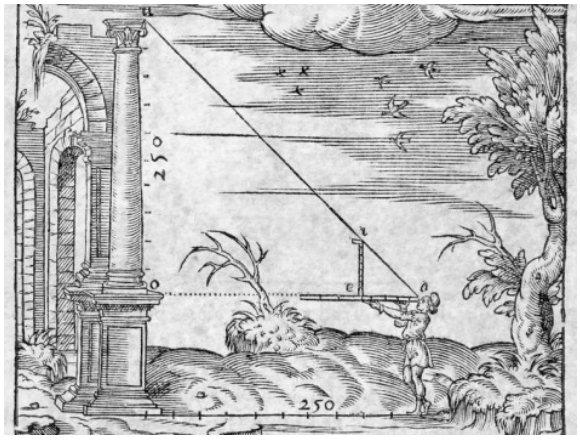
6.1 Basics of Trig Equations

Practice Tasks



I. Concepts and Procedures

1. Definition of Trigonometric Functions:

   Let t be any real number and let $P(x, y)$ be the terminal point on the unit circle determined by t.

   a. Define sine, cosine and tangent in terms of x and y.

   b. Define cosecant, secant and cotangent in two ways: in terms of sine, cosine and tangent, and in terms of x and y.

2. Using the definitions of the six trig functions and your knowledge of the common t values, fill in the following table:

Value of t	sin(t)	cos(t)	tan(t)	csc(t)	sec(t)	cot(t)
0						
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						
$\frac{\pi}{3}$						
$\frac{\pi}{2}$						
π						
$\frac{3\pi}{2}$						

3. Evaluate the following (use your knowledge of reference numbers to figure out problems with angles that are not in Quadrant I):

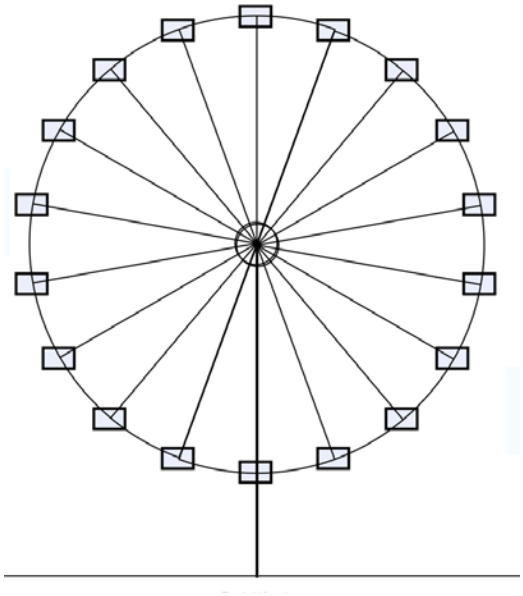
a. $\sin(\frac{5\pi}{6})$ b. $\tan(-\frac{\pi}{6})$ c. $\sec(\frac{7\pi}{4})$

d. $\csc(\frac{3\pi}{2})$ e. $\cos(\frac{13\pi}{6})$ f. $\cot(\frac{3\pi}{4})$

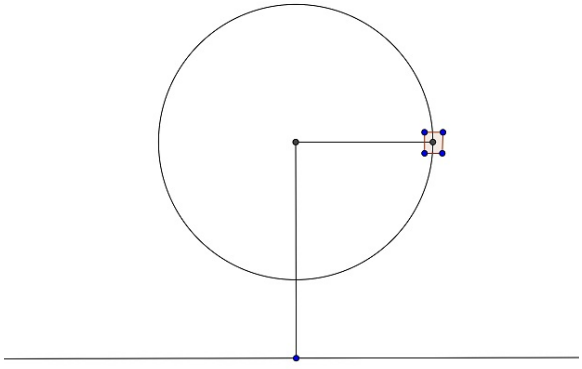
II. Problem Solving

1. Samantha, a mathematically minded student, is taking a trip to the carnival. While walking around, she notices that the sign next to the Ferris Wheel boasts that riders will reach 220 feet above ground while riding the Ferris Wheel. Sam is curious whether she can calculate her height at different points in the ride, and not just the highest point. She asks the attendant of the Ferris Wheel for more information, but all the man knows is that the central axle of the Ferris Wheel is 120 feet above the ground.

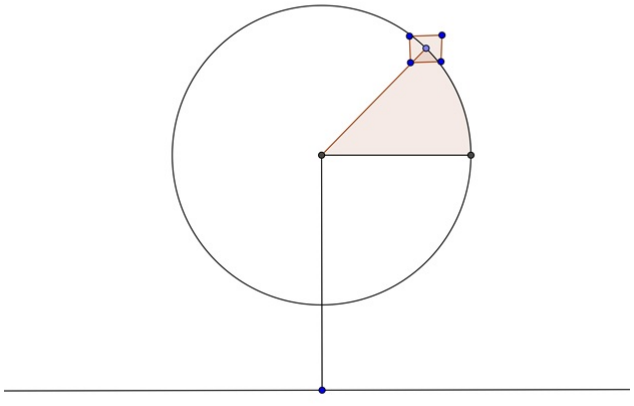
   a. Using the diagram below, draw in the measurements that you know, and use them to determine the radius of the Ferris Wheel. Also determine how far from the ground the Ferris Wheel will be at its lowest point.



b. Sam thinks of the Ferris Wheel as a very large Unit Circle. She realizes that there are two differences between the Unit Circle and the Ferris Wheel:
1) The Ferris Wheel is not centered at the origin and
2) the Ferris Wheel does not have a radius of 1.
At the starting position of the Unit Circle (when the angle equals 0 or 2π), what is the height of the Ferris Wheel?



c. Next, Sam looks at the Ferris Wheel when the car is halfway between the Unit Circle starting position (in question b.) and its highest position. A drawing of this is shown below. What is the angle measure, in radians, that the car travels between these two positions?

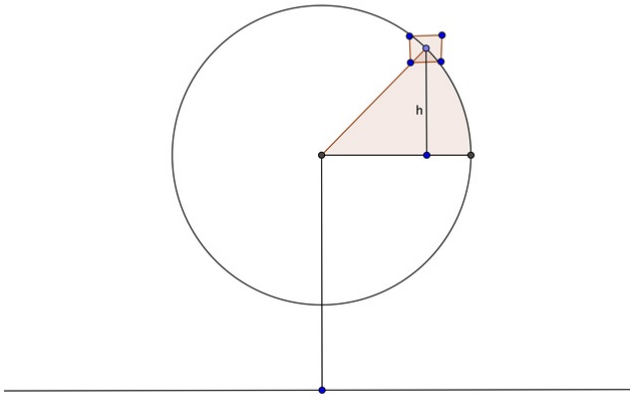


d. Having learned about Unit Circle Trigonometry in school, Sam knows that, by using the sine and cosine functions, she can find the x and y coordinates of the circle. Evaluate sine and cosine at the angle measurement you found in part c.

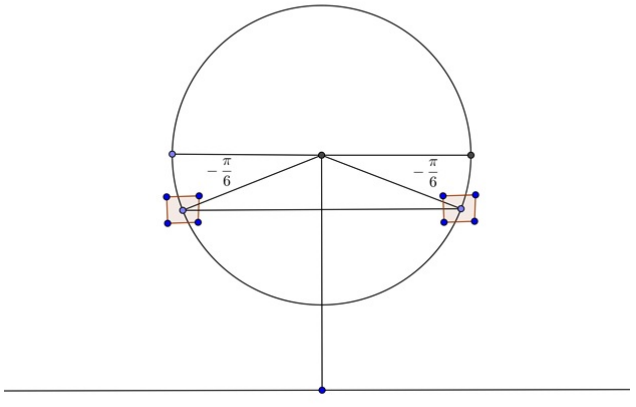
e. Sam is puzzled. The numbers she got in part d. were too small to make sense for the Ferris Wheel, so she asks her friend Luz for help. Luz chuckles, "Silly Sam, you found the height on the Unit Circle. Since the Ferris Wheel has a much larger radius than the Unit Circle, you need to multiply the sine and cosine values by the radius of the Ferris Wheel."

i. Use this fact to find the height of the car above the axel of the Ferris Wheel (labeled as h in the diagram below).

ii. Determine the height of the car off the ground.



f. For her final problem, Sam wants to challenge herself. She would like to find the distance between two cars that are exactly opposite from each other. Both of the cars are $\frac{\pi}{6}$ below the horizontal diameter (as shown below). Use trigonometry to determine the distance between the two cars.



III. Reasoning

1. Assume that some angle θ is in Quadrant II.
   a. What is the sign (positive or negative) of $\cos\theta$? $\sin\theta$?

   b. What is the sign of $\cos(-\theta)$? $\sin(-\theta)$?

   c. The cosine is an **even function**, which means that plugging in x or $-x$ (for any value of x) will give you the same result. Use your answers in part a and b to prove that cosine is an even function.

   d. Prove that the tangent is an **odd function**, such that $-\tan x = \tan(-x)$, for all values of x.

IV. Modeling

1. Now that you are a pro at finding different measurements of the Ferris Wheel, create and solve a new problem with a differently sized Ferris Wheel and a different number of seats. You may ask for the height of a car, the distance between two cars, or any other similar situation that requires trigonometry. Draw and label the Ferris Wheel below, along with your problem and solutions.