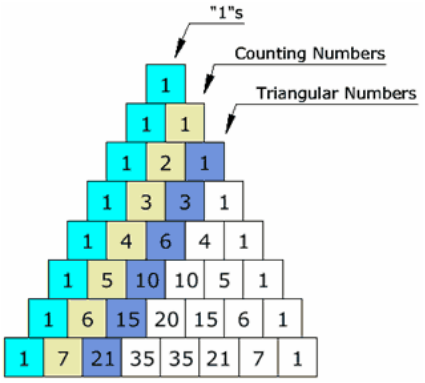
5.9 The Binomial Theorem

Practice Tasks



I. Concepts and Procedures

1. Evaluate the following expressions.
   a. $\frac{9!}{8!}$
   b. $\frac{7!}{5!}$
   c. $\frac{21!}{19!}$
   d. $\frac{8!}{4!}$

2. Use the binomial theorem to expand the following binomial expressions.
   a. $(x+y)^4$
   b. $(x+2y)^4$
   c. $(x+2xy)^4$
   d. $(x-y)^4$
   e. $(x-2xy)^4$

3. Use the binomial theorem to expand the following binomial expressions.
   a. $\left(1+\sqrt{2}\right)^5$
   b. $(1+i)^9$
   c. $(1-\pi)^5$ (Hint: $1-\pi = 1+(-\pi)$.)
   d. $\left(\sqrt{2}+i\right)^6$
   e. $(2-i)^6$

4. Consider the expansion of $(a+b)^{12}$. Determine the coefficients for the terms with the powers of a and b shown.
   a. a^2b^{10}
   b. a^5b^7
   c. a^8b^4

5. Consider the expansion of $(x+2y)^{10}$. Determine the coefficients for the terms with the powers of x and y shown.
   a. x^2y^8
   b. x^4y^6
   c. x^5y^5

6. Consider the expansion of $(5p+2q)^6$. Determine the coefficients for the terms with the powers of p and q shown.
   a. p^2q^4
   b. p^5q
   c. p^3q^3

7. Consider the binomial $(2u-3v)^6$.
   a. Find the term that contains v^4.
   b. Find the term that contains u^3.
   c. Find the third term.

8. Consider the binomial $(u^2-v^3)^6$.
   a. Find the term that contains v^6.
   b. Find the term that contains u^6.
   c. Find the fifth term.

9. Find the sum of all coefficients in the following binomial expansion.
   a. $(2u + v)^{10}$
   b. $(2u - v)^{10}$
   c. $(2u - 3v)^{11}$
   d. $(u - 3v)^{11}$
   e. $(1 + i)^{10}$
   f. $(1 - i)^{10}$
   g. $(1 + i)^{200}$
   h. $(1 + v)^{201}$

10. Expand the binomial $\left(1 + \sqrt{2}i\right)^6$.

11. Show that $\left(2 + \sqrt{2}i\right)^{20} + \left(2 - \sqrt{2}i\right)^{20}$ is an integer.

II. Reasoning

1. Explain why the coefficient of the term that contains u^n is 1 in the expansion of $(u+v)^n$.

2. Explain why the coefficient of the term that contains $u^{n-1}v$ is n in the expansion of $(u+v)^n$.

3. Explain why the rows of Pascal's triangle are symmetric. That is, explain why $C(n,k) = C\big(n,(n-k)\big)$.

4. We know $(u+v)^2 = u^2 + 2uv + v^2 = u^2 + v^2 + 2uv$. Use this pattern to predict what the expanded form of each expression would be. Then, expand the expression, and compare your results.
   a. $(u+v+w)^2$
   b. $(a+b+c+d)^2$

5. Look at the powers of 101 up to the fourth power on a calculator. Explain what you see. Predict the value of 101^5, and then find the answer on a calculator. Are they the same?

6. Can Pascal's triangle be applied to $\left(\frac{1}{u}+\frac{1}{v}\right)^n$ given $u, v \neq 0$?