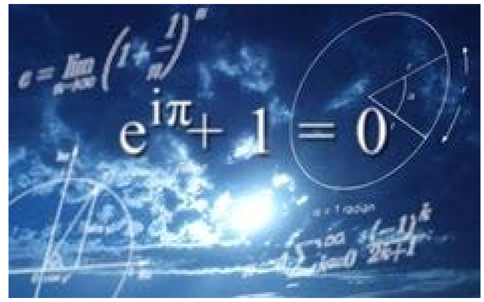
5.5 Polar Form of Complex Numbers

Practice Tasks



1. Concepts and Procedures

1. A complex number has two parts: a is the ____________________part, and b is the ________________ part. To graph , we graph the ordered pair (____ , _____) in the complex plane.

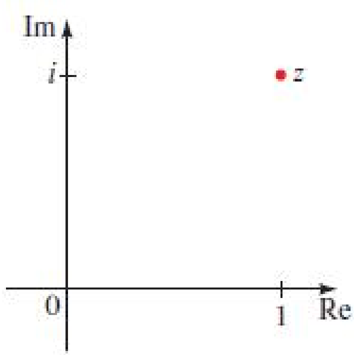
2. Let $z = a + bi$.

a. The modulus of z is $r =$ _____, and an argument of z is an angle satisfying $tan\theta =$ _____.

b. We can express z in polar form as ___________________, where r is the modulus of z and θ is the argument of z.

3. **a.** The complex number $z =$ -1 + i in polar form is $z =$ ________________________. The complex number $z = 2(cos\frac{\pi}{6} + i \cdot sin\frac{\pi}{6})$ in rectangular form is ________________________.

b. The complex number graphed below can be expressed in rectangular form as _____________________ or in polar form as ___________________________.



4. Graph the complex number and find its modulus and argument.

a. 4i

b. -2

c. 5 + 2i

d. $\sqrt{3} + i$

e. $\frac{3+4i}{5}$

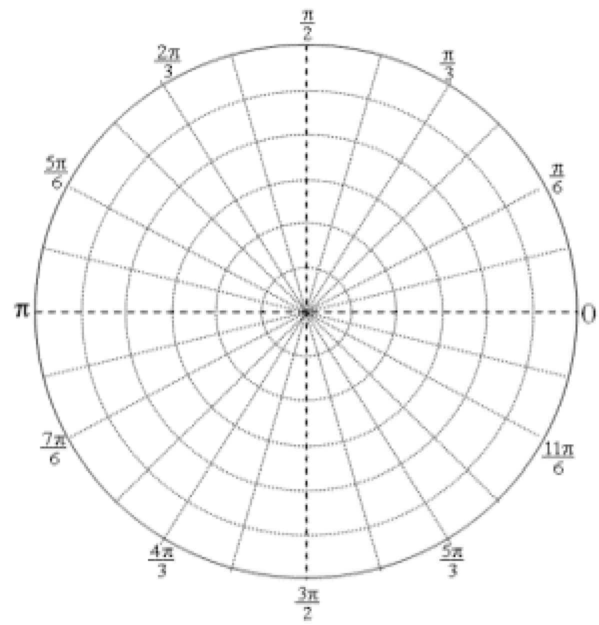
5. Graph each complex number on the polar grid. Label each point with capital letter. Then express it in rectangular form.

a. $4(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

b. $3(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

c. $\frac{3}{2}(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4})$

d. $2(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3})$



6. Write the complex number in polar form with argument θ between 0 and 2π.

a. $1 + i$

b. $1 - i$

c. $-3i$

d. 4

e. $i(2 - 2i)$

7. Find the product $z_1 \cdot z_2$. Express your answer in polar form.

a. $z_1 = \cos(\pi) + i \cdot \sin(\pi)$; $z_2 = \cos\left(\frac{\pi}{3}\right) + i \cdot \sin(\frac{\pi}{3})$

b. $z_1 = 3(\cos\left(\frac{\pi}{6}\right) + i \cdot \sin\left(\frac{\pi}{6}\right))$; $z_2 = 5(\cos\left(\frac{4\pi}{3}\right) + i \cdot \sin\left(\frac{4\pi}{3}\right))$

c. $z_1 = 4(\cos(120°) + i \cdot \sin(120°)$; $z_2 = 2(\cos(30°) + i \cdot \sin(30°)$

8. Write z1 and z2 in polar form. Find the product $z_1 \cdot z_2$ and the quotient $\frac{z_1}{z_2}$.

a. $z_1 = \sqrt{3} + i$; $z_2 = 1 + \sqrt{3}i$

b. $z_1 = 2\sqrt{3} - 2i$; $z_2 = -1 + i$

9. Find the indicated power using De Moivre's Theorem.

a. $(1 + i)^{20}$

b. $(2\sqrt{3} + 2i)^5$

c. $(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i)^{12}$

d. $(-1 - i)^7$

e. $(2\sqrt{3} + 2i)^{-5}$

10. Find the indicated roots, and graph the roots in the complex plane.

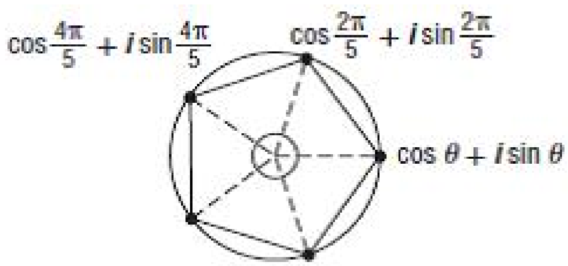
a. The square roots of $4\sqrt{3} + 4i$

b. The fourth roots of -81i.

c. The cube roots of i

II. Problem Solving

1. METAL WORKING Lindsay is casting a metal plate in the shape of a regular pentagon. To model the plate, she rotates the radius of a circle in increments of 72° to inscribe a regular pentagon in the circle and then plots the vertices in the plane as shown.



a. What is the measure of each angle of rotation in radians?

b. Find the two other vertices of the pentagon.

2. ELECTRICAL ENGINEERING In electrical circuits with alternating current, impedance is the opposition to the flow of current in a circuit. Most common electrical devices operate on a 115-volt, alternating current. The formula $I = \frac{E}{Z}$ describes the relationship among the current I, the voltage E, and the impedance Z. Find each quantity and express your answer in rectangular form.

a. Find the voltage if $I = 10(\cos 35° + i\sin 35°)$ and $Z = 3(\cos 20° + i\sin 20°)$.

b. Find the impedance if $I = 8(\cos 5° + i\sin 5°)$ and $E = 115(\cos 45° + i\sin 45°)$.

3. BICYCLES The overall tension of the spokes on a bicycle wheel affects the wheel's stiffness and its ability to absorb shock. Plucking the spokes and listening to the musical tone of the vibrating spoke is an indicator of tension. Suppose you pluck a spoke at a position described by a complex number $a + bi$.

a. If you move your fingers to a position represented by $2(a + bi)$ are they on the same spoke?

b. How is the distance your fingers are from the origin related to their original position?

4. FURNITURE DESIGN One corner of a tabletop is plotted on the complex plane at the origin. Two other corners are at $1 + 3i$ and $6 + 2i$. The third corner is at $(1 + 3i) + (6 + 2i)$.

a. Graph the tabletop.

b. Describe the shape of the tabletop.

c. Write $1 + 3i$ in polar form.

d. Find $(6 + 2i)^2$ in polar form.