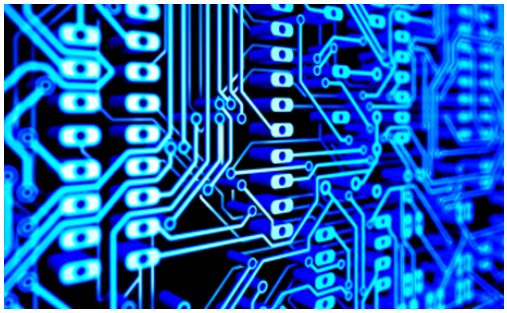
5.4 Distance on the Complex Plane

Practice Tasks



I. Concepts and Procedures

1. Find the midpoint between the two given points in the rectangular coordinate plane.
   a. $2 + 4i$ and $4 + 8i$

   b. $-3 + 7i$ and $5 - i$

   c. $-4 + 3i$ and $9 - 4i$

   d. $4 + i$ and $-12 - 7i$

   e. $-8 - 3i$ and $3 - 4i$

   f. $\frac{2}{3} - \frac{5}{2}i$ and $-0.2 + 0.4i$

2. Find the distance between the following points.
   a. Point $A(2, 3)$ and point $B(6, 6)$

   b. $A = 2 + 3i$ and $B = 6 + 6i$

   c. $A = -1 + 5i$ and $B = 5 + 11i$

   d. $A = 1 - 2i$ and $B = -2 + 3i$

   e. $A = \frac{1}{2} - \frac{1}{2}i$ and $B = -\frac{2}{3} + \frac{1}{3}i$

II. Problem Solving

1. Given three points A, B, C, where C is the midpoint of A and B.
   a. If $A = -5 + 2i$ and $C = 3 + 4i$, find B.

   b. If $B = 1 + 11i$ and $C = -5 + 3i$, find A.

2. Point C is the midpoint between $A = 4 + 3i$ and $B = -6 - 5i$. Find the distance between points C and D for each point D provided below.
   a. $2D = -6 + 8i$

   b. $D = -\bar{B}$

3. The distance between points $A = 1 + i$ and $B = a + bi$ is 5. Find the point B for each value provided below.
   a. $a = 4$

   b. $b = 6$

III. Reasoning

1. Let $A = 2 + 4i$, $B = 14 + 8i$, and suppose that C is the midpoint of A and B, and that D is the midpoint of A and C.

   a. Find points C and D.

   b. Find the distance between A and B.

   c. Find the distance between A and C.

   d. Find the distance between C and D.

   e. Find the distance between D and B.

   f. Find a point one quarter of the way along the line segment connecting segment A and B, closer to A than to B.

g. Terrence thinks the distance from B to C is the same as the distance from A to B. Is he correct? Explain why or why not.

h. Using your answer from part (g), if E is the midpoint of C and B, can you find the distance from E to C? Explain.

i. Without doing any more work, can you find point E? Explain.

IV. Modeling

1. Draw five points in the plane A, B, C, D, E. Start at any position, P_0, and leapfrog over A to a new position, P_1 (so, A is the midpoint of $\overline{P_0P_1}$). Then leapfrog over B, then C, then D, then E, then A, then B, then C, then D, then E, then A again, and so on. How many jumps will it take to get back to the start position, P_0?

2. For the leapfrog puzzle problems in both Exploratory Challenge 1 and Problem 5, we are given an odd number of points to leapfrog over. What if we leapfrog over an even number of points? Let $A = 2$, $B = 2 + i$, and $P_0 = i$. Will P_n ever return to the starting position, P_0? Explain how you know.