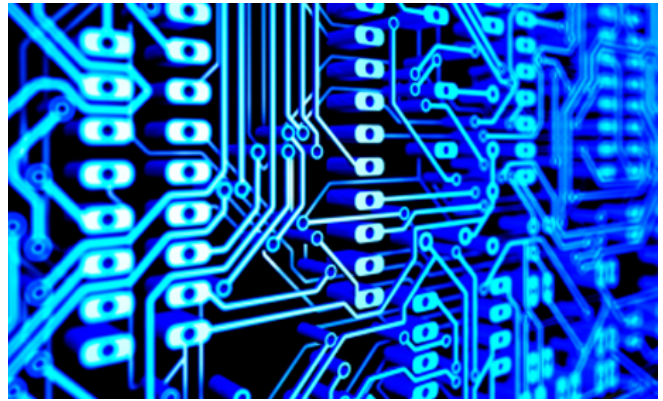


## 5.4: Distance on the Complex Plane

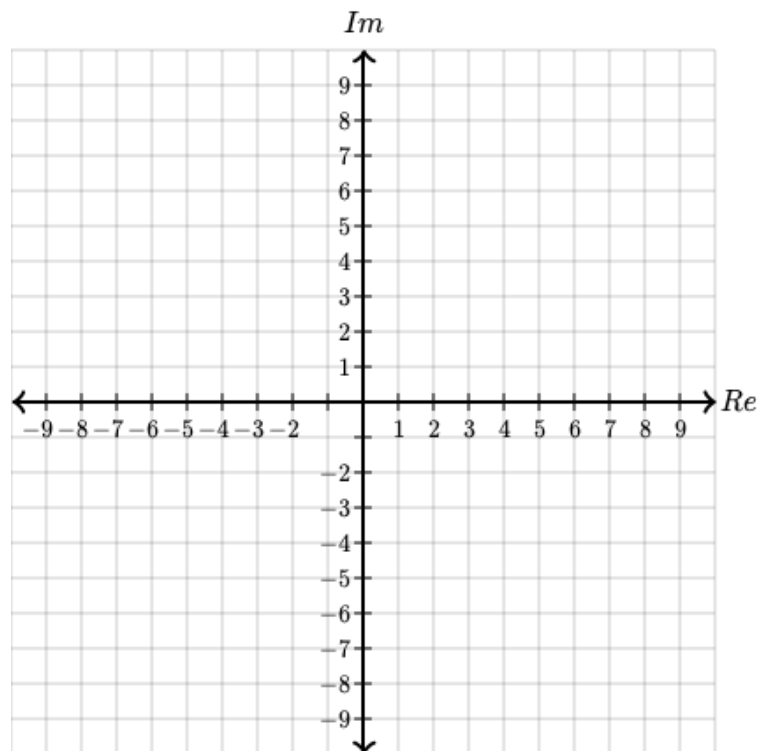
### *Distance and Midpoint Formulae*

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In this lesson, you will learn how to calculate distance and midpoints on the complex plane... and how to play Leap Frog!

1. In this investigation, you will compute some values related to the complex numbers  $2+i$  and  $5-3i$ .
  - a. Plot  $2+i$  and  $5-3i$  in the complex plane.
  - b. How far is  $2+i$  from  $5-3i$ ? That is, what is the length of the line segment between  $2+i$  and  $5-3i$ ? (Show your work.)

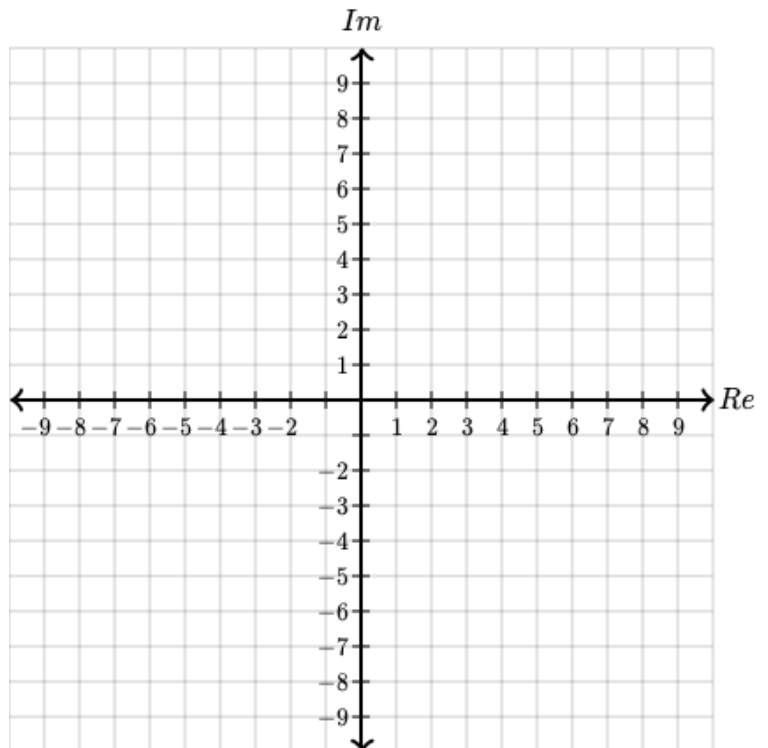


- a. What is the modulus of the difference of  $2+i$  and  $5-3i$ ?
- b. What is the midpoint of the line segment between  $2+i$  and  $5-3i$ ?

- e. What is the average of  $2+i$  and  $5-3i$ ?
- f. Describe, using the complex plane, the relationship between your answers in (b) and (c).
- g. Describe, using the complex plane, the relationship between your answers in (d) and (e).

Practice Your Skills:

2. The endpoints of  $\overline{AB}$  are  $A(1,8)$  and  $B(-5,3)$ . What is the midpoint of  $\overline{AB}$ ?
3. What is the midpoint of  $A = 1 + 8i$  and  $B = -5 + 3i$ ?
4. Using  $A = x_1 + y_1i$  and  $B = x_2 + y_2i$ , show that, in general, the midpoint of points  $A$  and  $B$  is  $\frac{A+B}{2}$ , the arithmetic average of the two numbers.



5. The endpoints of  $\overline{AB}$  are  $A(1,8)$  and  $B(-5,3)$ . What is the length of  $\overline{AB}$ ?
  
6. What is the distance between  $A = 1 + 8i$  and  $B = -5 + 3i$ ?
  
7. Show that, in general, the distance between  $A = x_1 + y_1i$  and  $B = x_2 + y_2i$  is the modulus of  $A - B$ .

## II. Game Challenge: *Complex Leapfrog*

Work with a partner (or two) in the following activity. First you will figure out a secret formula for finding an endpoint of a segment when given one endpoint and the midpoint (task 8b). Then you will play the game (task 9). You will need a large piece of graph paper for the game.

8. Let  $A = 2 + 3i$  and  $B = -4 - 8i$ . Find a complex number  $C$  so that  $B$  is the midpoint of  $A$  and  $C$ .
  - a. Given two complex numbers  $A$  and  $B$ , find a formula for a complex number  $C$  in terms of  $A$  and  $B$  so that  $B$  is the midpoint of  $A$  and  $C$ .
  
  - b. Verify that your formula is correct by using the result of part (a).
  
9. Complex Leap Frog – Challenge 1
  - a. Draw three points  $A$ ,  $B$ , and  $C$  in the plane.
  
  - b. Start at any position  $P_0$  and leapfrog over  $A$  to a new position  $P_1$  so that  $A$  is the midpoint of  $\overline{P_0P_1}$ .
  - c. From  $P_1$ , leapfrog over  $B$  to a new position  $P_2$  so that  $B$  is the midpoint  $\overline{P_1P_2}$ .
  - d. From  $P_2$ , leapfrog over  $C$  to a new position  $P_3$  so that  $C$  is the midpoint  $\overline{P_2P_3}$ .
  - e. Continue alternately leapfrogging over  $A$ , then  $B$ , then  $C$ .

- f. What eventually happens?
- g. Using the formula from Question 11 part (b), show why this happens.

#### 10. Leap Frog Challenge 2

- a. Plot a single point  $A$  in the plane.
- b. What happens when you repeatedly jump over  $A$ ?
- c. Using the formula from Opening Exercise part (b), show why this happens.
- d. Make a conjecture about what will happen if you leapfrog over two points,  $A$  and  $B$ , in the coordinate plane.
- e. Test your conjecture by using the formula from Opening Exercise part (b).
- f. Was your conjecture correct? If not, what is your new conjecture about what happens when you leapfrog over two points,  $A$  and  $B$ , in the coordinate plane?
- g. Test your conjecture by actually conducting the experiment.