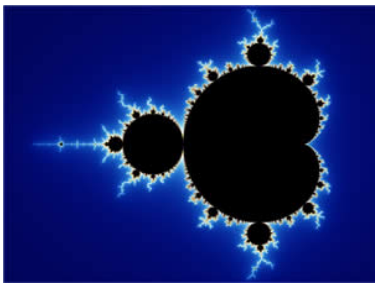
5.3 Complex Multiplication

Practice Tasks



I. Concepts and Procedures

1. The _____ of a complex number is when you switch the sign of the complex part.

2. Find the complex conjugate of the following:
   a. $-5 + 3i$

   b. $4i$

   c. $1.23 + 2.73i$

3. Given the complex numbers $w = 2 - 3i$ and $z = -3 + 2i$, graph each of the following:

   a. $w - 2$

   b. $z + 2$

   c. $w + 2i$

   d. $z - 3i$

   e. $w + z$

   f. $z - w$

4. Let $z = -4 + 2i$, simplify the following and describe the geometric effect of the operation. Plot the result in the complex plane.
   a. $z + 2 - 3i$

   b. $z - 2 - 3i$

   c. $z - (2 - 3i)$

   d. $2z$

   e. $\frac{z}{2}$

5. Let $z = -1 + 2i, w = 4 - i$, simplify the following expressions.
   a. $z + \overline{w}$

   b. $|w - \bar{z}|$

   c. $2z - 3w$

   d. $\frac{z}{w}$

6. Let $z = 1 + 2i$, simplify the following and describe the geometric effect of the operation.
   a. iz

   b. $i^2 z$

   c. $\bar{z}$

   d. $-\bar{z}$

   e. $i\bar{z}$

   f. $2iz$

   g. $iz + 5 - 3i$

7. Simplify the following expressions.

   a. $(4-2i)(5-3i)$

   b. $(-2+3i)(-2-3i)$

   c. $(1+i)^2$

   d. $(1+i)^{10}$ [Hint: $b^{nm}=(b^n)^m$]

   e. $\frac{-1+2i}{1-2i}$

   f. $\frac{x^2+4}{x-2i}$, provided $x \neq 2i$.

8. Given $z=2+i$, describe the geometric effect of the following. Plot the result.

   a. $z(1+i)$

   b. $z\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

II. Reasoning

1. Given the complex number z, find a complex number z + w where z + w is shifted
   a. $2\sqrt{2}$ in a northeast direction

   b. $5\sqrt{2}$n a southeast direction

2. We learned that multiplying by i produces a 90° counterclockwise rotation about the origin. What do we need to multiply by to produce a 90° clockwise rotation about the origin?

III. Modeling

1. Let $w =$ the complex number of your own choosing. Then find z for each case.
   a. z is a 270° counterclockwise rotation about the origin of w.

   b. z is reflected over the imaginary axis from w.

   c. z is reflected over the real axis from w.