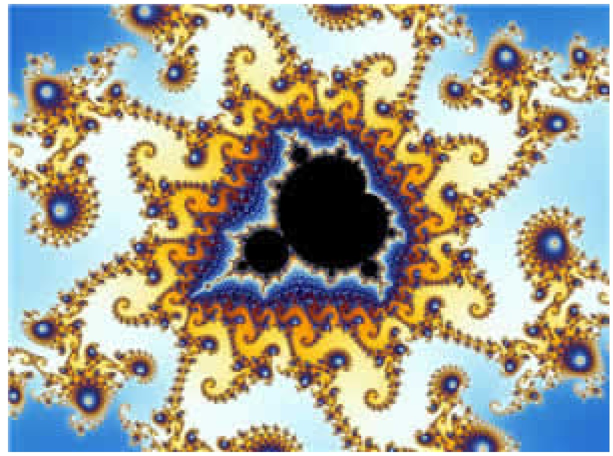
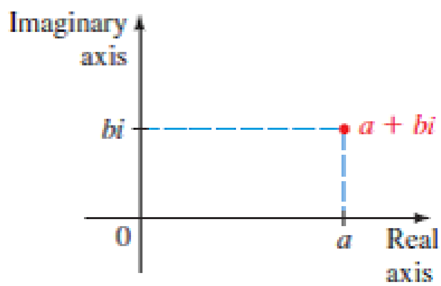
5.2: Complex Graphing

Geometric Effects of Complex Arithmetic



Complex numbers can be interpreted geometrically by plotting them on the complex plane.

To graph real numbers or sets of real numbers, we have been using the number line, which has just one dimension. Complex numbers, however, have two components: a real part and an imaginary part. This suggests that we need two axes to graph complex numbers: one for the real part and one for the imaginary part. We call these the **real axis** and the **imaginary axis**, respectively. The plane determined by these two axes is called the **complex plane.**

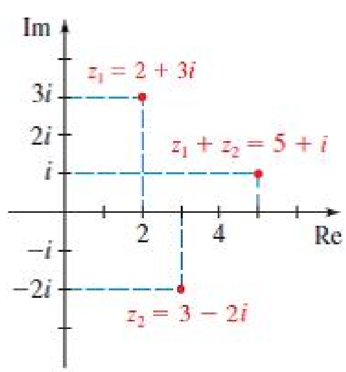


I . Graphing on the Complex Plane

To graph the complex number $a+bi$, we plot the ordered pair of numbers (a, b) in this plane, as illustrated below.

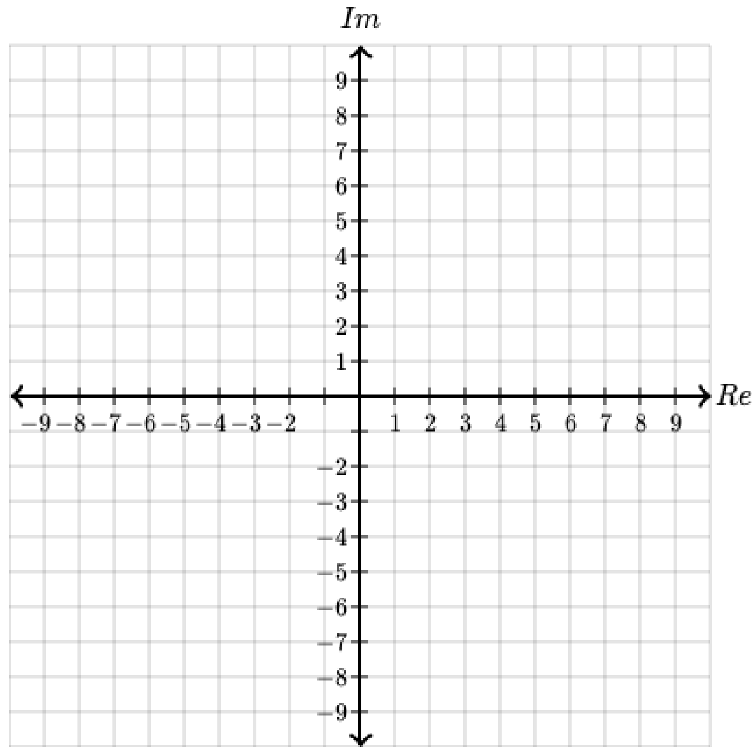
EXAMPLE 1: Graph the complex numbers $z_1=2+3i$, $z_2=3-2i$ and z_1+z_2

The graph is shown below:



<u>Your Turn</u>:

1. Let $z = a + bi$, where a and b are real numbers. Plot the following complex numbers on the complex plane.



a. $z_1 = 3 + 2i$

b. $z_2 = 1 + 2i$

c. $z_3 = -3 - 2i$

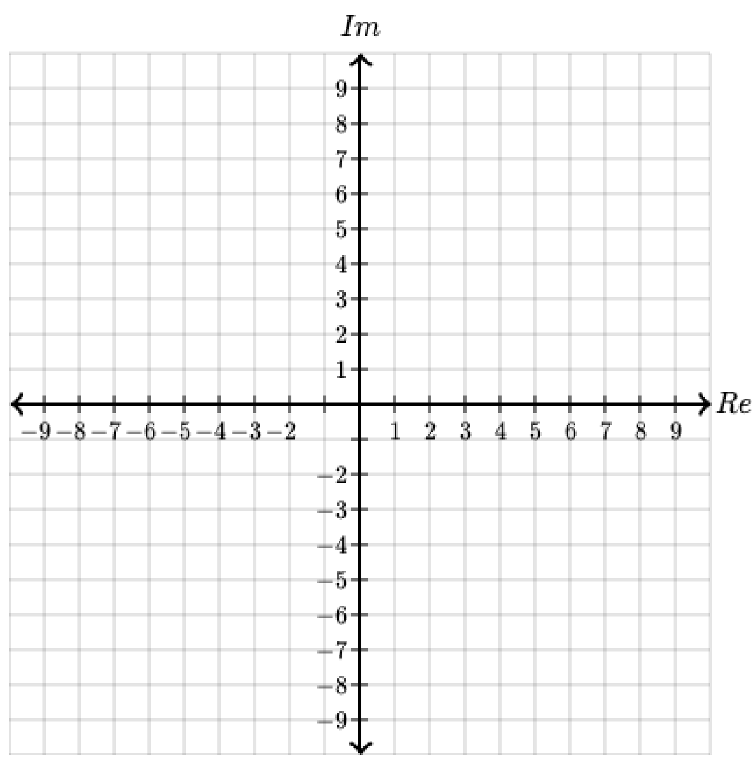
d. $z_1 + z_2$

e. . $z_2 + z_3$

f. . $z_1 + z_3$

II. The Geometric Effect of *Some* Complex Arithmetic

2. Find the conjugate, and plot the complex number *and* its conjugate in the complex plane. Label the conjugate with a prime symbol (A', etc).



   a. A: $6 + 3i$

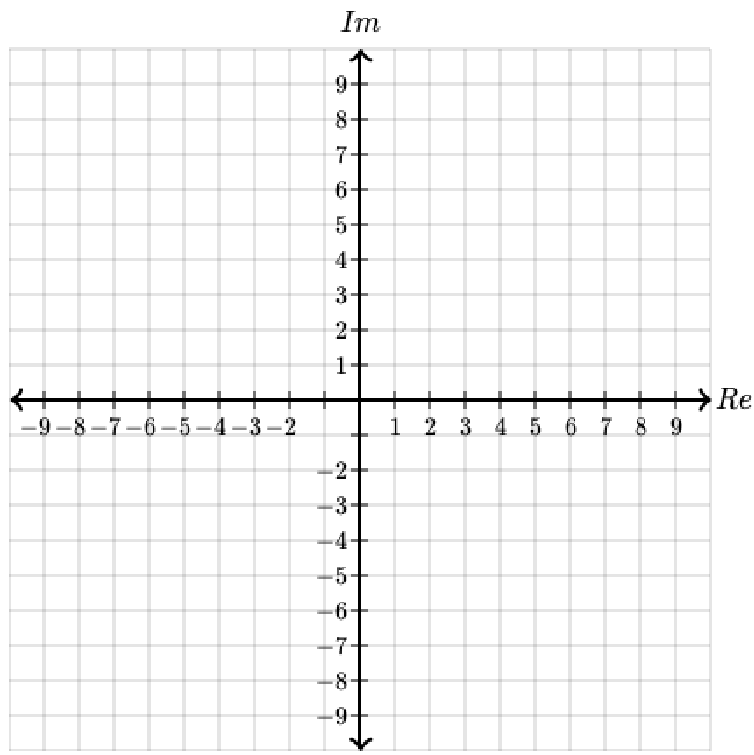
   b. B: $-4 + 5i$

   c. C: $8i$

3. Describe the <u>geometric effect</u> of taking the conjugate of a complex number.

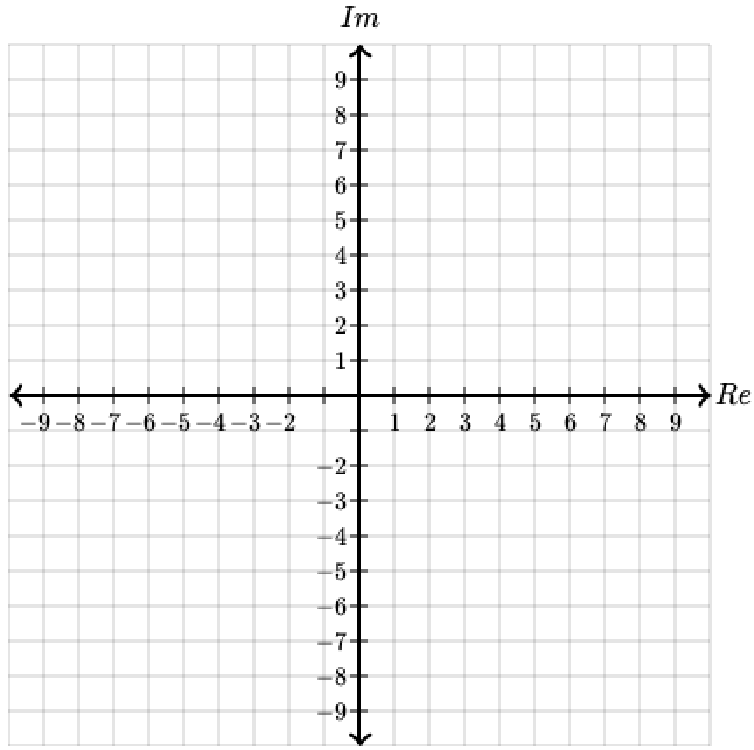
4. What operation on a complex number induces a reflection across the imaginary axis? (Experiment, if you need.)

5. Given the complex numbers $w = -4 + 3i$ and
   $z = 2 - 5i$,
   graph each of the following:
   a. w
   b. z
   c. $w + 2$
   d. $z + 2$
   e. $w - 1$
   f. $z - 1$



6. Describe in your own words the geometric effect adding or subtracting a real number has on a complex number.

7. Given the complex numbers $w = -4 + 3i$ and $z = 2 - 5i$, graph each of the following:
    g. w
    h. z
    i. $w + i$
    j. $z + i$
    k. $w - 2i$
    l. $z - 2i$



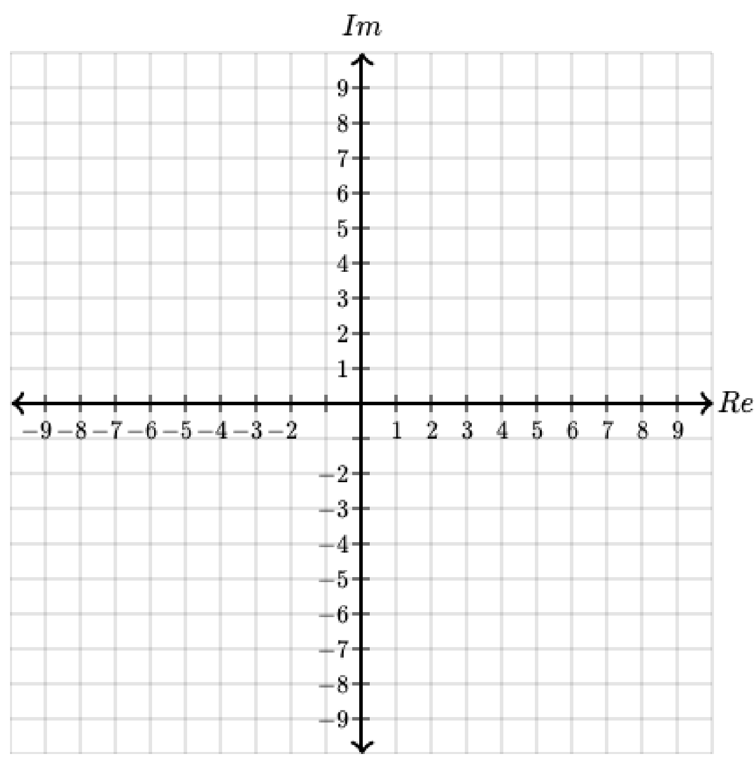
8. Describe in your own words the geometric effect adding or subtracting an imaginary number has on a complex number.

9. Given $z = 3 - 2i$, plot and label the following and describe the geometric effect of the operation.

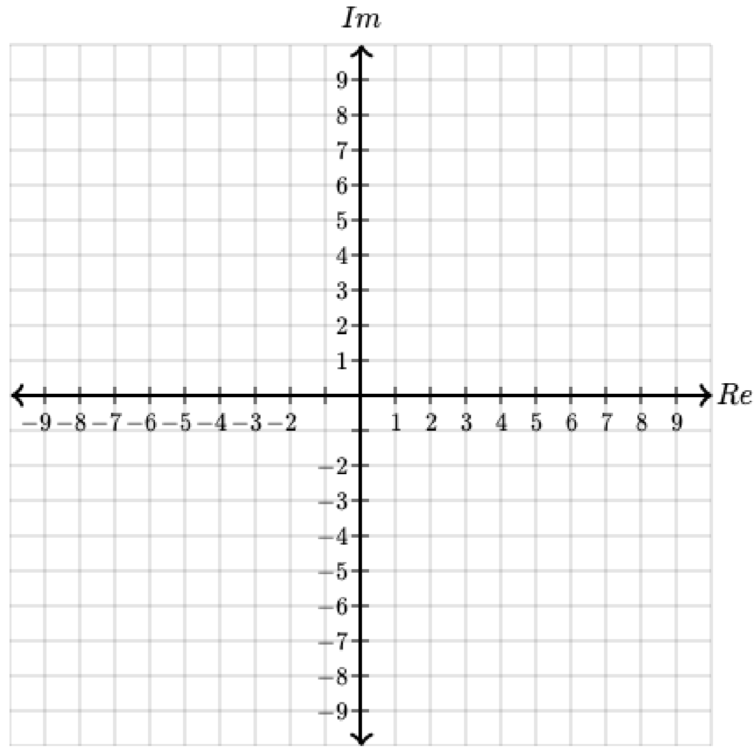
    a. $z - 2$

    b. $z + 4i$

    c. $z + (-2 + 4i)$

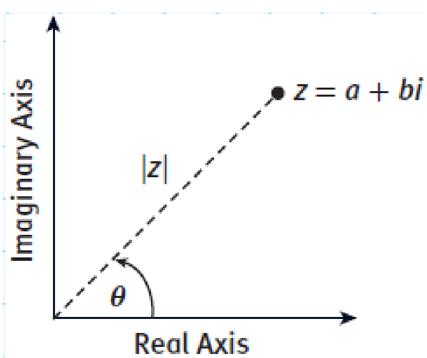


10. Given the complex number z, find a complex number w such that $z + w$ is shifted $\sqrt{2}$ units in a *southwestern* direction.



III. The Modulus and Argument of a Complex Number

There are two definitions that are important to the geometric interpretation of complex numbers: the **modulus** and the **argument.**



The *modulus* (or **absolute value)** of a complex number $z = a + bi$, written as $|a + bi|$ or $|z|$, gives the **distance** from zero to $a + bi$ on the complex plane (the dashed line).

An *argument* of a complex number, written $\theta = \arg(z)$, is an **angle** formed by the positive real axis and the line segment connecting zero to z.

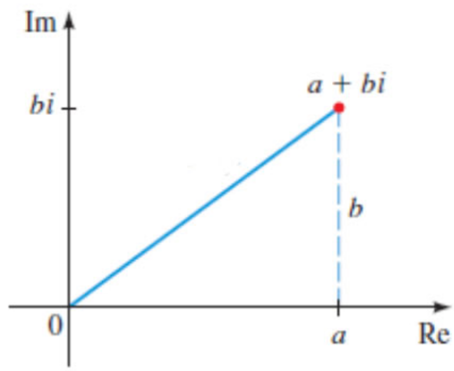
The figure above illustrates both themodulus of z, $|z|$, and the argument of z, $\arg(z)$.

> Note: Arguments of a complex number are not unique. To simplify calculations in this activity, let the argument of a complex number be the smallest positive angle whose terminal side is the segment between zero and z.

11. Write an expression, in terms of *a* and *b*, to determine the value of the argument of *z*, or θ. [Hint: use trig to find the angle θ.]

12. Write an expression, in terms of *a* and *b*, to determine the value of $|z|$.

Hint: use diagram below:



EXAMPLE 2: Finding the Modulus and Argument of a Complex Numbers

Find the Modulus of complex number $3+4i$

$\sqrt{a^2+b^2}$	Formula for Modulus
$\sqrt{3^2+4^2}$	Substitute for *a* and *b*
$\sqrt{25}$	Combine real and imaginary parts
10 – 2i	ANSWER (modulus)

Find an Argument of complex number $3+4i$

$\tan^{-1}\frac{b}{a}$	Formula for Argument
$\tan^{-1}\frac{4}{3}$	Substitute for *a* and *b*
$\approx 53.13°$	ANSWER (argument)

Your Turn:

13. Find the modulus and the argument of each complex number below.

a. $z = 2i$

b. $z = 5+2i$

c. $z = 2 - i$ (Careful – this is not in the first quadrant!)

d. $z = -3 - 2i$