4.8 Determinants and Cramer's Rule
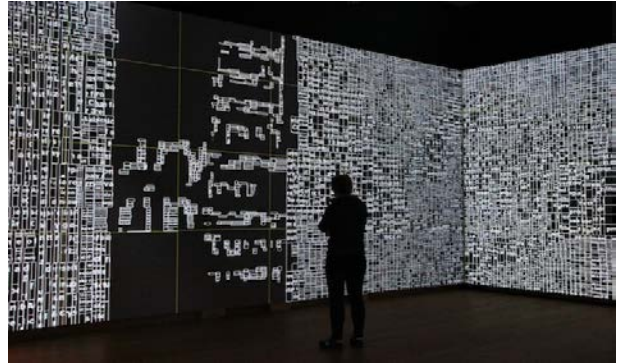
Practice Tasks

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1. Find the determinant of the matrix. Determine whether the matrix has an inverse, but don't calculate the inverse.

    a. $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

    b. $\begin{bmatrix} 4 & 5 \\ 0 & -1 \end{bmatrix}$

    c. $\begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & 4 \\ 0 & 1 & -3 \end{bmatrix}$

    d. $\begin{bmatrix} 30 & 0 & 20 \\ 0 & -10 & -20 \\ 40 & 0 & 10 \end{bmatrix}$

    e. $\begin{bmatrix} 1 & 3 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ -1 & 0 & 0 & 2 \\ 1 & 6 & 4 & 1 \end{bmatrix}$

    f. $\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 0 & 3 & 6 & 9 \\ 0 & 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 0 & 5 \end{vmatrix}$

2. Use Cramer's Rule to solve the system.

a.
$$\begin{cases} 2x - y = -9 \\ x + 2y = 8 \end{cases}$$

b.
$$\begin{cases} 0.4x + 1.2y = 0.4 \\ 1.2x + 1.6y = 3.2 \end{cases}$$

c.
$$\begin{cases} x - y + 2z = 0 \\ 3x + z = 11 \\ -x + 2y = 0 \end{cases}$$

d.
$$\begin{cases} x + y + z + w = 0 \\ 2x + w = 0 \\ y - z = 0 \\ x + 2z = 1 \end{cases}$$

3. Sketch the triangle with the given vertices, and use a determinant to find its area.

a. (0,0), (3,2), (6.8)

b. (-1,3), (2,9), (5-6)

II. **Problem Solving**

1. A roadside fruit stand sells apples at 75¢ a pound, peaches at 90¢ a pound, and pears at 60¢ a pound. Muriel buys 18 pounds of fruit at a total cost of $13.80. Her peaches and pears together cost $1.80 more than her apples.
   a. Set up a linear system for the number of pounds of apples, peaches, and pears that she bought.
   b. Solve the system using Cramer's Rule.

2. A hair product company sells three types of hair products for $30, $20, and $10 per unit. In one year, the total revenue for the three products was $800,000, which corresponded to the sale of 40,000 units. The company sold half as many units of the $30 product as units of the $20 product. Use Cramer's Rule to solve a system of linear equations to find how many units of each product were sold.

3. An outdoors club is purchasing land to set up a conservation area. The last remaining piece they need to buy is the triangular plot shown in the figure. Use the determinant formula for the area of a triangle to find the area of the plot.
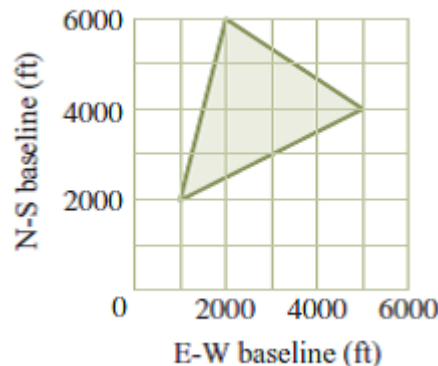
   a. Find the coordinates of the vertices of the surrounding rectangle, and find its area.
   b. Find the area of the red triangle by subtracting the areas of the three blue triangles from the area of the rectangle.
   c. Use your answer to part (b) to show that the area of the red triangle is given by

$$\text{area} = \pm\tfrac{1}{2} \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$