4.7 Doing Business with Matrices

Practice Tasks

I. Concepts and Procedures

1. a. Write the following system as a matrix equation $AX = B$.

System	Matrix equation
	$A \cdot X = B$
$5x + 3y = 4$ $3x + 2y = 3$	$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}\begin{bmatrix} \square \\ \square \end{bmatrix} = \begin{bmatrix} \square \\ \square \end{bmatrix}$

b. The inverse of A is $A^{-1} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

c. The solution of the matrix equation is $X = A^{-1}B$.

$$\begin{array}{ccccc} X & = & A^{-1} & B & \\ \begin{bmatrix} x \\ y \end{bmatrix} & = & \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} & \begin{bmatrix} \square \\ \square \end{bmatrix} & = \begin{bmatrix} \square \\ \square \end{bmatrix} \end{array}$$

d. The solution of the system is x = _________ and y = ____________.

2. Solve the system of equations by converting to a matrix equation and using the inverse of the coefficient matrix.

a. $\begin{cases} 2x + 5y = 2 \\ -5x - 13y = 20 \end{cases}$

b. $\begin{cases} 2x + 4y + z = 7 \\ -x + y - z = 0 \\ x + 4y = -2 \end{cases}$

3. Use a calculator that can perform matrix operations to solve the system of equations.

a. $$\begin{cases} 12x + \frac{1}{2}y - 7z = 21 \\ 11x - 2y + 3z = 43 \\ 13x + y - 4z = 29 \end{cases}$$

b. $$\begin{cases} x + y + z + w = 15 \\ x - y + z - w = 5 \\ x + 2y + 3z + 4w = 26 \\ x - 2y + 3z - 4w = 2 \end{cases}$$

4. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 7 \\ 3 & -4 \end{bmatrix}$, $C = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$, $Z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Evaluate the following:

a. $A + B$
b. $B + A$
c. $A + (B + C)$
d. $(A + B) + C$
e. $A + I$
f. $A + Z$
g. $A \cdot Z$
h. $Z \cdot A$
i. $I \cdot A$
j. $A \cdot B$
k. $B \cdot A$
l. $A \cdot C$
m. $C \cdot A$
n. $A \cdot B + A \cdot C$
o. $A \cdot (B + C)$
p. $A \cdot B \cdot C$
q. $C \cdot B \cdot A$
r. $A \cdot C \cdot B$

II. Problem Solving

1. A nutritionist is studying the effects of the nutrients folic acid, choline, and inositol. He has three types of food available, and each type contains the following amounts of these nutrients per ounce.

	Type A	Type B	Type C
Folic acid (mg)	3	1	3
Choline (mg)	4	2	4
Inositol (mg)	3	2	4

   a. Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 2 & 4 \end{bmatrix}$ and use it to solve the remaining parts of this problem.

      i. How many ounces of each food should the nutritionist feed his laboratory rats if he wants their daily diet to contain 10 mg of folic acid, 14 mg of choline, and 13 mg of inositol?

   b. How much of each food is needed to supply 9 mg of folic acid, 12 mg of choline, and 10 mg of inositol?

   c. Will any combination of these foods supply 2 mg of folic acid, 4 mg of choline, and 11 mg of inositol?

2. A pet-store owner feeds his hamsters and gerbils different mixtures of three types of rodent food: KayDee Food, Pet Pellets, and Rodent Chow. He wishes to feed his animals the correct amount of each brand to satisfy their daily requirements for protein, fat, and carbohydrates exactly. Suppose that hamsters require 340 mg of protein, 280 mg of fat, and 440 mg of carbohydrates, and gerbils need 480 mg of protein, 360 mg of fat, and 680 mg of carbohydrates each day. The amount of each nutrient (in mg) in one gram of each brand is given in the following table. How many grams of each food should the storekeeper feed his hamsters and gerbils daily to satisfy their nutrient requirements?

	KayDee Food	Pet Pellets	Rodent Chow
Protein (mg)	10	0	20
Fat (mg)	10	20	10
Carbohydrates (mg)	5	10	30

3. A coffee manufacturer sells a 10-pound package that contains three flavors of coffee for $26. French vanilla coffee costs $2 per pound, hazelnut flavored coffee costs $2.50 per pound, and Swiss chocolate flavored coffee costs $3 per pound. The package contains the same amount of hazelnut as Swiss chocolate. Let represent the number of pounds of French vanilla, represent the number of pounds of hazelnut, and represent the number of pounds of Swiss chocolate.
   a. Write a system of linear equations that represents the situation.
   b. Write a matrix equation that corresponds to your system.
   c. Solve your system of linear equations using an inverse matrix. Find the number of pounds of each flavor of coffee in the 10-pound package.

4. A florist is creating 10 centerpieces for the tables at a wedding reception. Roses cost $2.50 each, lilies cost $4 each, and irises cost $2 each. The customer has a budget of $300 allocated for the centerpieces and wants each centerpiece to contain 12 flowers, with twice as many roses as the number of irises and lilies combined.
   a. Write a system of linear equations that represents the situation.
   b. Write a matrix equation that corresponds to your system.
   c. Solve your system of linear equations using an inverse matrix. Find the number of flowers of each type that the florist can use to create the 10 centerpieces.

5. An encyclopedia saleswoman works for a company that offers three different grades of bindings for its encyclopedias: standard, deluxe, and leather. For each set that she sells, she earns a commission based on the set's binding grade. One week she sells one standard, one deluxe, and two leather sets and makes $675 in commission. The next week she sells two standard, one deluxe, and one leather set for a $600 commission. The third week she sells one standard, two deluxe, and one leather set, earning $625 in commission.
   a. Let x, y, and z represent the commission she earns on standard, deluxe, and leather sets, respectively. Translate the given information into a system of equations in x, y, and z.
   b. Express the system of equations you found in part (a) as a matrix equation of the form $AX = B$.

c. Find the inverse of the coefficient matrix A and use it to solve the matrix equation in part (b). How much commission does the saleswoman earn on a set of encyclopedias in each grade of binding?

III. Reasoning

1. A 2×2 matrix of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ is a **diagonal matrix.** Daniel calculated

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & -6 \end{bmatrix}$$
$$\begin{bmatrix} 2 & 3 \\ 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & -6 \end{bmatrix}$$

and concluded that if X is a diagonal matrix and A is any other matrix, then $X \cdot A = A \cdot X$.

Is there anything wrong with Daniel's reasoning? Prove or disprove that if X is a diagonal 2×2 matrix, then $X \cdot A = A \cdot X$ for any other matrix A.

For 3×3 matrices, Elda claims that only diagonal matrices of the form $X = \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}$ satisfy $X \cdot A = A \cdot X$ for any other 3×3 matrix A.

Is her claim correct?

IV. Modeling

1. Create an activity that models the job of a **purchasing manager**. Find a simple recipe for chocolate chip cookies. Find as many prices for each ingredient as possible and find the cost for each measure of each ingredient from each different source. Place these in matrices. Try to find the least expensive recipe for chocolate chip cookies and the source of each ingredient and the most expensive recipe for chocolate chip cookies and the source of each ingredient. Compare your results to other members of your class.