4.7: Doing Business with Matrices

Matrix Equations and More Cryptography

Hamm, Her, and Nales Construction Company builds houses. They build three different styles of houses in urban development project. Shown below is a table for the cost associated with constructing houses in three cities.

The company has been asked to submit a bid to the City of Beijing to build 60 Ranch, 12 Cape and 48 Two-Family houses.

Driving Question: How much will it cost to build the desired number of houses?

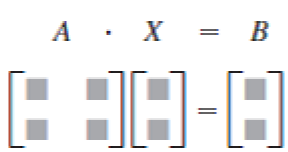
	Ranch	Cape	2-Family	Cost (dollars)
Guangzhou	7	29	16	13,450,000
Shanghai	16	10	35	16,200,000
Tianjin	12	0	48	16,800,000

1. Write a system of equations that represents the data in the table.

2. In terms of the construction project, what are x, y and z?

3. Write a matrix equation that describes the problem. Record it under the template.

    a. Write a 3 x 3 matrix A that represents the number of houses sold
    b. Write a 3 x 1 matrix X that represents the three unknown quantities
    c. Write a 3 x 1 matrix B that represents the money earned in the three cities



II. Solving Matrix Equations

Solving the matrix equation $AX = B$ is very similar to solving the simple real-number equation

$$3x = 12$$

which we do by multiplying each side by the reciprocal (or inverse) of 3.

$$\frac{1}{3} \cdot 3x = \frac{1}{3} \cdot 12$$

$$x = 4$$

We solve the matrix equation by multiplying each side by the inverse of A (provided that this inverse exists):

$AX = B$	
$A^{-1}(AX) = A^{-1}B$	Multiply on left by A^{-1}
$(A^{-1}A)X = A^{-1}B$	Associative Property
$I_3X = A^{-1}B$	Property of inverses
$X = A^{-1}B$	Property of identity matrix

So to solve the Hamm, Her & Nales constructiion problem all you have to do is find the inverse of matrix A (if it exists) and multiply that by the matrix.

4. Use a calculator that utilizes matrix operations to solve the problem
   a. Find A^{-1}

   b. Multiply $A^{-1}B$

   c. State the solution in the context of the problem.

   d. How much would it cost the company to build 60 Ranch, 12 Cape and 48 Two-Family houses in Beijing?

EXAMPLE: Solving Matrix Equations

To solve a system of equations using n x n matrices:

$$\begin{cases} x - 2y - 4z = 7 \\ 2x - 3y - 6z = 5 \\ -3x + 6y + 15z = 0 \end{cases}$$

- First, rewrite the system as a matrix equation

$$\underbrace{\begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 7 \\ 5 \\ 0 \end{bmatrix}}_{B}$$

- Find the inverse of matrix A

$$A^{-1} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -\frac{2}{3} \\ 1 & 0 & \frac{1}{3} \end{bmatrix}$$

- Multiply $A^{-1}B$

$$\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -\frac{2}{3} \\ 1 & 0 & \frac{1}{3} \end{bmatrix}}_{A^{-1}} \underbrace{\begin{bmatrix} 7 \\ 5 \\ 0 \end{bmatrix}}_{B} = \begin{bmatrix} -11 \\ -23 \\ 7 \end{bmatrix}$$

SOLUTION: x = -11, y = 23, and z = 7 (Use units and context from real-life situations.)

5. Solve the system of equations by converting to a matrix equation and using the inverse of the coefficient matrix.

a. $\begin{cases} -3x - 5y = 4 \\ 2x + 3y = 0 \end{cases}$

b. $\begin{cases} x + y - 2z = 3 \\ 2x + 5z = 11 \\ 2x + 3y = 12 \end{cases}$

III. Cryptography Challenge

You have been assigned a group number. The message your group receives is listed below. This message is TOP SECRET! It is of such importance that it has been encoded four times.

Your group's portion of the coded message is listed below.

- Group 1:

  $1500, 3840, 0, 3444, 3420, 4350, 0, 4824, 3672, 3474, -2592, -6660, 0, -5976, -5940, -7560, 0, -8388, -6372, -6048$

- Group 2:

  $2424, 3024, -138, 396, -558, -1890, -1752, 1512, -2946, 1458, 438, 540, -24, 72, -90, -324, -300, 270, -510, 270$

- Group 3:

  $489, 1420, 606, 355, 1151, 33, 1002,$
  $829, 99, 1121, 180, 520, 222, 130, 422, 12, 366, 304, 36, 410$

- Group 4:

  $-18, 10, -18, 44, -54, 42, -6, -74, -98, -124, 0, 10, -12, 46, -26, 42, -4, -36, -60, -82$

- Group 5:

  $-120, 0, -78, -54, -84, -30, 0, -6, -108, -30, -120,$
  $114, 42, 0, -12, 42, 0, 36, 0, 0$

- Group 6:

  $126, 120, 60, 162, 84, 120, 192, 42, 84, 192, -18, -360,$
  $-90, -324, 0, -18, -216, -36, -90, -324$

6. Store your message in a matrix C with two rows. How many columns does matrix C have?

7. Begin at the station of your group number, and apply the decoding matrix at this first station.

8. Proceed to the next station in numerical order; if you are at Station 6, proceed to Station 1. Apply the decoding matrix at this second station.

9. Proceed to the next station in numerical order; if you are at Station 6, proceed to Station 1. Apply the decoding matrix at this third station.

10. Proceed to the next station in numerical order; if you are at Station 6, proceed to Station 1. Apply the decoding matrix at this fourth station.

11. Decode your message.

12. Assemble the messages from all six teams.