4.6 Cryptography & Matrices

Practice Tasks

I. Concepts and Procedures

1. a. The matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called an ________________________ matrix.

   b. If A is a 2 x 2 matrix, then $A \times I =$ _______ and $I \times A =$ ___________.

   c. If A and B are 2 x 2 matrices with $AB = I$, then B is the ________________ of A.

2. Calculate the products AB and BA to prove that B is the inverse of A.

   a. $A = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$

   b. $A = \begin{vmatrix} 1 & 3 & -1 \\ 1 & 4 & 0 \\ -1 & -3 & 2 \end{vmatrix}$ $\quad B = \begin{vmatrix} 8 & -3 & 4 \\ -2 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix}$

3. Find the inverse and verify that $A^{-1}A = AA^{-1} = I_2$ and $B^{-1}B = BB^{-1} = I_3$

   a. $A = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$

   b. $B = \begin{vmatrix} 1 & 3 & 2 \\ 0 & 2 & 2 \\ -2 & -1 & 0 \end{vmatrix}$

4. Find the determinant, then find the inverse matrix if it exists:

   a. $\begin{bmatrix} -3 & -5 \\ 2 & 3 \end{bmatrix}$

   b. $\begin{bmatrix} 2 & 5 \\ -5 & -13 \end{bmatrix}$

c. $\begin{bmatrix} 6 & -3 \\ -8 & 4 \end{bmatrix}$

d. $\begin{bmatrix} 0.4 & -1.2 \\ 0.3 & 0.6 \end{bmatrix}$

e. $\begin{vmatrix} 2 & 4 & 1 \\ -1 & 1 & 1 \\ 1 & 4 & 0 \end{vmatrix}$

f. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \\ 1 & -1 & -10 \end{vmatrix}$

g. $\begin{vmatrix} 0 & -2 & 2 \\ 3 & 1 & 3 \\ 1 & -2 & 3 \end{vmatrix}$

h. $\begin{vmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 0 & 2 \end{vmatrix}$

II. Problem Solving

1. Decode the message below using the matrix $D = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$:

   22, 17, 24, 9, −1, 14, −9, 34, 44, 64, 47, 77.

2. You received a coded message in the matrix $C = \begin{bmatrix} 30 & 30 & 69 \\ 2 & 1 & 15 \\ 9 & 14 & 20 \end{bmatrix}$. However, the matrix D that will decode this message has been corrupted, and you do not know the value of entry d_{12}. You know that all entries in matrix D are integers. Using x to represent this unknown entry, the decoding matrix D is given by $D = \begin{bmatrix} 2 & x & -4 \\ -1 & 2 & 3 \\ 1 & -1 & -2 \end{bmatrix}$. Decode the message in matrix C.

III. Reasoning

1. Claire claims that she multiplied $A = \begin{bmatrix} -3 & 2 \\ 0 & 4 \end{bmatrix}$ by another matrix X and obtained $\begin{bmatrix} -3 & 2 \\ 0 & 4 \end{bmatrix}$ as her result. What matrix did she multiply by? How do you know?

2. Brandon encoded his name with the matrix $E = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, producing the matrix $C = \begin{bmatrix} 6 & 33 & 15 & 14 \\ 12 & 66 & 30 & 28 \end{bmatrix}$. Decode the message, or explain why the original message cannot be recovered.

3. Janelle used the encoding matrix $E = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$ to encode the message "FROG" by multiplying
$C = \begin{bmatrix} 6 & 18 \\ 15 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 24 & 30 \\ 22 & 37 \end{bmatrix}$. When Taylor decoded it, she computed
$M = \begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 24 & 30 \\ 22 & 37 \end{bmatrix} = \begin{bmatrix} 20 & 44 \\ 2 & -7 \end{bmatrix}$. What went wrong?

4. Show that the only matrix B such that $A + B = A$ is the zero matrix.

IV. Modeling

1. Create a secret code that uses groups of four blocks of characters and a 4 x 4 matrix encryption key.