4.5 Linear Systems & Matrices

Practice Tasks

I. Concepts and Procedures

1. If a system of linear equations has infinitely many solutions, then the system is called ________________________________. If a system of linear equations has no solution, then the system is called __________________________.

2. Write the augmented matrix of the following system of equations.

$$\begin{cases} x + y - z = 1 \\ x + 2z = -3 \\ 2y - z = 3 \end{cases}$$

3. The following matrix is the augmented matrix of a system of linear equations in the variables *x*, *y*, and z. (It is given in reduced row-echelon form.)

$$\begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   a. The leading variables are _____________________.
   b. Is the system inconsistent or dependent? Explain how you know.
   c. The solution of the system is:
   $x =$ ___________ $y =$ ___________ $z =$ ___________

4. The augmented matrix of a system of linear equations is given in reduced row-echelon form. Find the solution of the system.

(a) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

$x =$ _____ $x =$ _____ $x =$ _____

$y =$ _____ $y =$ _____ $y =$ _____

$z =$ _____ $z =$ _____ $z =$ _____

5. Solve the system of linear equations using Gaussian Elimination.

a. $$\begin{cases} x - 2y + z = 1 \\ y + 2z = 5 \\ x + y + 3z = 8 \end{cases}$$

b. $$\begin{cases} x + y + z = 2 \\ 2x - 3y + 2z = 4 \\ 4x + y - 3z = 1 \end{cases}$$

6. Solve the system of linear equations using Gauss-Jordan Elimination.

a. $$\begin{cases} 4x - 3y + z = -8 \\ -2x + y - 3z = -4 \\ x - y + 2z = 3 \end{cases}$$

b. $$\begin{cases} 2x + y + 3z = 9 \\ -x - 7z = 10 \\ 3x + 2y - z = 4 \end{cases}$$

c. $$\begin{cases} x + 2y - 3z = -5 \\ -2x - 4y - 6z = 10 \\ 3x + 7y - 2z = -13 \end{cases}$$

d. $$\begin{cases} x - y + 6z = 8 \\ x + z = 5 \\ x + 3y - 14z = -4 \end{cases}$$

II. **Problem Solving**

1. A doctor recommends that a patient take 50 mg each of niacin, riboflavin, and thiamin daily to alleviate a vitamin deficiency. In his medicine chest at home the patient finds three brands of vitamin pills. The amounts of the relevant vitamins per pill are given in the table. How many pills of each type should he take every day to get 50 mg of each vitamin?

	VitaMax	Vitron	VitaPlus
Niacin (mg)	5	10	15
Riboflavin (mg)	15	20	0
Thiamin (mg)	10	10	10

2. A furniture factory makes wooden tables, chairs, and armoires. Each piece of furniture requires three operations: cutting the wood, assembling, and finishing. Each operation requires the number of hours (h) given in the table. The workers in the factory can provide 300 hours of cutting, 400 hours of assembling, and 590 hours of finishing each work week. How many tables, chairs, and armoires should be produced so that all available labor-hours are used? Or is this impossible?

	Table	Chair	Armoire
Cutting (h)	$\frac{1}{2}$	1	1
Assembling (h)	$\frac{1}{2}$	$1\frac{1}{2}$	1
Finishing (h)	1	$1\frac{1}{2}$	2

III. Reasoning

1. The augmented matrix below represents a system of linear equations (in variables x, y, and z) that has been reduced using Gauss-Jordan elimination. Write a system of equations with nonzero coefficients that is represented by the reduced matrix.

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

IV. Modeling

1. We all know that two points uniquely determine a line $y = ax^2 + bx + c$ in the coordinate plane. Similarly, three points uniquely determine a quadratic (second-degree) polynomial $y = ax^3 + bx^2 + cx + d$, four points uniquely determine a cubic (third-degree) polynomial and so on. (Some exceptions to this rule are if the three points actually lie on a line, or the four points lie on a quadratic or line, and so on.)

   For the following set of five points, find the line that contains the first two points, the quadratic that contains the first three points, the cubic that contains the first four points, and the fourth-degree polynomial that contains all five points.
   (0,0), (1,12), (2,40), (3,6), (-1, -14)

   Graph the points and functions in the same viewing rectangle using a graphing device.