4.4 Matrix Properties

Practice Tasks

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I.   Concepts and Procedures

1. We can add (or subtract) two matrices only if they have the same _____.

2. a. We can multiply two matrices only if the number of in the first matrix is the same as the number of in the second matrix.

    b. If *A* is a 3 x 3 matrix and *B* is a 4 x 3 matrix, which of the following matrix multiplications are possible?

    (i) AB   (ii) BA   (iii) AA   (iv) BB

3. Which of the following operations can we perform for a matrix *A* of any dimension?
    (i) A + A   (ii) 2A   (iii) $A \cdot A$

4. Perform the matrix operation, or if it is impossible, explain why.

    a. $\begin{bmatrix} 2 & 1 & 2 \\ 6 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 3 & 6 \\ -2 & 0 \end{bmatrix}$

    b. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}$

    c. $\begin{bmatrix} 2 & -3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

5. The matrices A, B, C, D, E, F, G and H are defined as follows.

$$A = \begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & \frac{1}{2} & 5 \\ 2 & -1 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -\frac{5}{2} & 0 \\ 0 & 2 & -3 \end{bmatrix} \qquad D = \begin{bmatrix} 7 & 3 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} 5 & -3 & 10 \\ 6 & 1 & 0 \\ -5 & 2 & 2 \end{bmatrix} \qquad H = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

Carry out the indicated operation, or explain why it cannot be performed.

a. B + C

b. C – B

c. 5A

d. 3B + 2C

e. AD

f. DH

g. AH

h. BC
i. GF

j. B^2

k. A^2

l. (DA)B

m. ABE

n. DB + DC

III. **Problem Solving**
1. A small fast-food chain with restaurants in Santa Monica, Long Beach, and Anaheim sells only hamburgers, hot dogs, and milk shakes. On a certain day, sales were distributed according to the following matrix.

Number of items sold

	Santa Monica	Long Beach	Anaheim
Hamburgers	4000	1000	3500
Hot dogs	400	300	200
Milk shakes	700	500	9000

$= A$

The price of each item is given by the following matrix.

Hamburger	Hot dog	Milk shake
[$0.90	$0.80	$1.10] $= B$

a. Calculate the product AB.

b. Interpret the entries in the product matrix AB.

2. Different point values are awarded for different shots in basketball. Create matrices from the information below.

Shot	Point(s)
Free Throw	1
2-pointer	2
3-pointer	3

Player	Free Throws	2-pointer	3-pointer
Tasha	44	32	25
Emily	37	24	31
Maritza	35	39	29

a. Calculate the product AB.

b. Interpret the entries in the product matrix AB.

3. A specialty-car manufacturer has plants in Auburn, Biloxi, and Chattanooga. Three models are produced, with daily production given in the following matrix.

Cars produced each day

	Model K	Model R	Model W	
Auburn	12	10	0	
Biloxi	4	4	20	= A
Chattanooga	8	9	12	

Because of a wage increase, February profits are lower than January profits. The profit per car is tabulated by model in the following matrix.

	January	February	
Model K	$1000	$500	
Model R	$2000	$1200	= B
Model W	$1500	$1000	

a. Calculate *AB.*

b. Assuming that all cars produced were sold, what was the daily profit in January from the Biloxi plant?

c. What was the total daily profit (from all three plants) in February?

4. ION Restaurant wishes to expand their business and is considering four possible options. If they build a new store they expect to make a profit of 7 million dollars if the market remains strong; however, if the market declines, they could have a loss of 5 million dollars. If ION invests in a franchise, they could profit 13 million dollars in a strong market but lose 7 million dollars in a declining market. If they

modernize their current facilities, they could profit 6 million dollars in a strong market but lose 3 million dollars in a declining one. If they sell their business, they will make a profit of 8 million dollars irrespective of the state of the market.

a. Write down a 4×2 payoff matrix P summarizing the profits and losses the company could expect to see with all possible scenarios. (Record a loss as a profit in a negative amount.)

b. The company realized that all their figures need to be adjusted by 7.5% in magnitude due to inflation costs. What is the value of a real number x to use as a scalar, to calculate the adjusted payoff matrix? Write down the new payoff matrix R.

IV. Reasoning

1. If AB has dimensions 5 x 8, with A having dimensions 5 x 6, what are the dimensions of B?

2. What must be true about the dimensions of the matrices A and B if both products AB and BA are defined?

V. Modeling

1. Two examples of the uses of matrices are given in Part II. Find another situation in real life in which parabolas occur, and create a math problem and solution that relates to that usage.