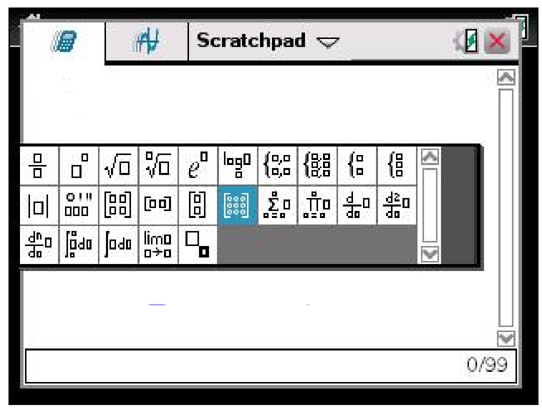
4.4: The Matrix Online

Discovering Matrix Properties using Graphing Calculator Technology

In the last lesson, you learned how to multiply a matrix by a scalar number. In this lesson you will learn how to use calculator technology to do complex arithmetic tasks… and to discover interesting properties of matrix multiplication.

I. Matrix Arithmetic with the TI-nSpire

- Press the Home Button and choose **Calculator**.
- To access the (m x n) matrix template (pictured on the right):
  - Press the **Template Key** (to the right of the number 9 key.)
  - Highlight the small block, which pictures a 3 x 3 matrix (highlighted on the right)
  - Press **Enter**
  - Enter the number of rows *and* columns and press **Enter**.
- The handheld displays an empty matrix. Move to each element in the matrix and type the appropriate value in each cell.



1. Find the answers to the following matrix arithmetic problems:

   a. $\begin{bmatrix} 1 & \frac{3}{7} & 2 \\ 3 & 4 & \frac{5}{12} \end{bmatrix} + \begin{bmatrix} \frac{5}{3} & \frac{2}{3} & 1 \\ \frac{2}{5} & 2 & \frac{1}{7} \end{bmatrix} =$

   b. $\begin{bmatrix} 2.378 & -5.32 \\ 12.3 & 29 \end{bmatrix} - \begin{bmatrix} -3.25 & 1.6 \\ -3 & 15.79 \end{bmatrix} =$

   c. $-0.376 \cdot \begin{bmatrix} 1 & 2 \\ -1.32 & 3 \\ 43 & -0.38 \\ -12 & 15.37 \end{bmatrix} =$

If you need to work repeatedly with the same matrix, it saves time to store the matrix as a variable.

To store the matrix as a variable, press the right arrow until you exit the matrix, press **CTRL** and **Store As** , type the name of the matrix, and press **Enter**.

To retrieve the stored matrix, press **Var** and select the variable.

$$A = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 7 & -3 & 0 \\ 0 & 1 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 6 & 0 & -6 \\ 8 & 1 & \frac{3}{2} \end{bmatrix}$$

2. Store the matrices above and perform the indicated operations on them:
   a. A + B

   b. C – D

   c. 5A – 3B

   d. C + A

   e. Explain why any of the calculations could not be performed.

3. Investigating Addition and Scalar Multiplication Properties: Use the stored matrices from Question 2 to answer the following questions:
   a. Is addition of matrices <u>Commutative</u>?
      <u>Hint</u>: Is C + D = C + D? Check with matrices A and B, as well.

   b. Is scalar multiplication <u>Commutative</u>?

   c. Is scalar multiplication <u>Distributive</u>?

II. Discovering Properties of Matrix Multiplication with the TI-nSpire

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 6 & 5 \\ 2 & 3 & 4 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 6 & 5 \\ 8 & 2 & 9 \\ 4 & 7 & 3 \end{bmatrix}$$

$$E = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \qquad F = \begin{bmatrix} 5 & 9 & 2 \\ 4 & 3 & 7 \\ 8 & 7 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} 6 & 9 & 5 & 5 \\ 8 & 3 & 9 & 7 \\ 2 & 7 & 3 & 1 \end{bmatrix}$$

4. Store the matrices above and perform the indicated operations on them, if they can be performed.

   Note: on the nSpire you must use the multiplication symbol to calculate matrix products, ex. $(A \cdot B)$ will calculate, but not AB.

   a. AB

   b. AC

   c. AD

   d. BC

   e. BD

5. Make a conjecture about the conditions required for matrix multiplication. (<u>Hint</u>: look at the dimensions.)

6. If matrix A has dimensions 5×2 and matrix B has dimensions 2×4, is it possible to multiply A times B? Explain your answer.

   a. What will be the dimensions of the product AB?

7. Use the stored matrices and perform the indicated operations on them:
    a. BE
    b. EB
    c. DF
    d. FD
8. Is multiplication of one matrix by another <u>Commutative</u>? Explain how your answers to Question 7 support your claim.
9. Use the stored matrices to investigate the Distributive Property for Matrix Multiplication.
    <u>Note</u>: Again, be sure to use the multiplication symbol in front of the parentheses, ex. $(A \cdot (B+E))$ will calculate, but not $A(B+E)$.
    a. Does $A \cdot (B+E) = AB + AE$?
    b. Does $C \cdot (D+F) = CD + CF$?
    c. Is multiplication of one matrix by another Distributive? Explain your reasoning.
10. Use the stored matrices to investigate the Associative Property for Matrix Multiplication.
    a. Is multiplication of one matrix by another Associative?
    b. Explain your reasoning, including the examples that support your reasoning.

III. Solidify Your Understanding Task

11. Josh and Jamilla have placed an order for lunch at a local fast-food restaurant. Josh ordered two cheeseburgers, two orders of fries, and a chocolate shake. Jamilla ordered one cheeseburger, one order of fries and one chocolate shake.

The information for each item follows:

Item	Cost	Calories	Fat
Cheeseburger	$2	300	12
Fries	$1	380	19
Shake	$3	580	21

a. Set up a 2×3 matrix that represents the cost, calories, and fat of the items (A).

b. Set up a 3×3 matrix that represents each person's order. (Store as matrix B.)

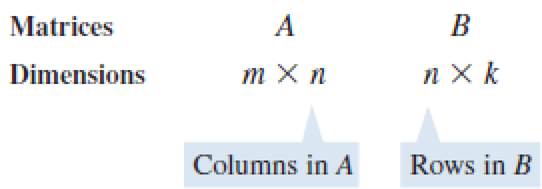
c. Use a calculator to compute the product AB.

d. Label the rows and columns in your product matrix AB.

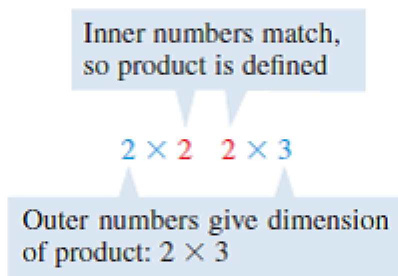
e. The labels for the rows of matrix AB come from Matrix ______ and the labels for the columns come from matrix _____.

f. Why is it important that the number of rows in matrix A match the number of columns in matrix B?

As witnessed in the Application, the two *inner numbers* must match when we multiply matrices…



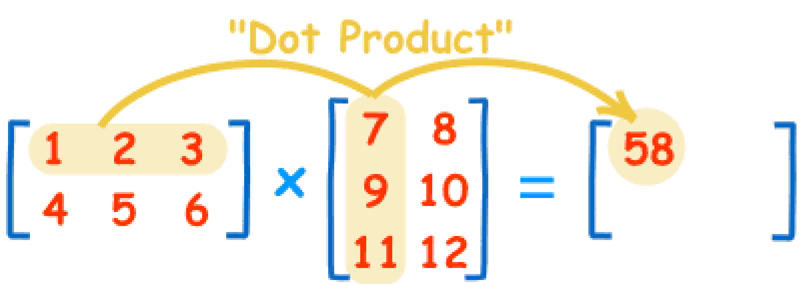
… and the two *outer numbers* give the dimensions of the product matrix.



12. If matrix A has dimensions 13 × 7 and matrix B has dimensions 7 × 5, is it possible to multiply A times B? Explain your answer.

    a. What will be the dimensions of the product AB?

When we multiply a matrix by another matrix we need to do the dot product of rows and columns ... what does that mean? Let us see with an example:



13. Complete the remaining elements of the product matrix above.

IV. Payoff Matrices

Game Theory is an advanced mathematics field that studies strategic decision making. It has applications in both the real world of business decisions and the virtual world of role play. One of the most useful tools in game theory is the payoff matrix.

14. Complete the following table that describes the different outcomes in the game *Rock Paper Scissors*.
    Remember: Rock beats Scissors, Scissors beats Paper, and Paper beats Rock.
    Note: +1 represents a player winning; -1 represents a player losing.

Player 2 (columns) / **Player 1** (rows)

	Rock	Paper	Scissor
Rock	0, 0	-1, 1	
Paper			
Scissor			

15. Create a payoff matrix for the game *Rock Paper Scissors*.

16. ION Restaurant is considering two parcels of land to build their new restaurant – one in West Hartford and another in East Hartford. The value of both parcels depends on the location of a new stadium being built: the West parcel will increase by 22 million dollars, if the stadium is built nearby, but it will lose 20 million dollars in value, if the stadium is built in East Hartford.

The table below represents the four options: buying the East parcel, the West parcel, both, or neither.

Restaurant Land Purchase Decision

Land Purchased at Location(s)	Stadium is Built at Location East	Stadium is Built at Location West
East	$22	($20)
West	($14)	$19
East and West	$8	($1)
Neither	$0	$0

Note: use negative numbers for losses [-14] instead of parentheses (14).

a. Create a 4×2 payoff matrix R summarizing the profits and losses the company expects from all possible scenarios.

b. A new assessment reveals that land values are now running 15% higher than when their plans were first created. What is the value of a real number x to use as a scalar, to calculate the adjusted payoff matrix? Write down the new payoff matrix Q.

PROPERTIES OF ADDITION AND SCALAR MULTIPLICATION OF MATRICES

Let A, B, and C be $m \times n$ matrices and let c and d be scalars.

$A + B = B + A$	Commutative Property of Matrix Addition
$(A + B) + C = A + (B + C)$	Associative Property of Matrix Addition
$c(dA) = cdA$	Associative Property of Scalar Multiplication
$(c + d)A = cA + dA$ $c(A + B) = cA + cB$	Distributive Properties of Scalar Multiplication