4.2 Graphs & Matrices

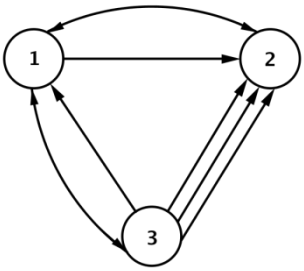
Practice Tasks

I. Concepts and Procedures

1. A _____ is an array of numbers organized into m rows and n columns.

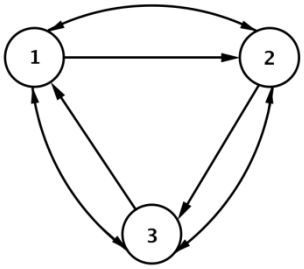
2. A matrix containing 5 rows and 3 columns has a _____ of 5×3.

3. Consider the railroad map between Cities 1, 2, and 3, as shown.



   a. Create a matrix R to represent the railroad map between Cities 1, 2, and 3.
   b. How many different ways can you travel from City 1 to City 3 without passing through the same city twice?
   c. How many different ways can you travel from City 2 to City 3 without passing through the same city twice?
   d. How many different ways can you travel from City 1 to City 2 with exactly one connecting stop?
   e. Why is this not a reasonable network diagram for a railroad?

4. Consider the subway map between stations 1, 2, and 3, as shown.



   a. Create a matrix S to represent the subway map between stations 1, 2, and 3.
   b. How many different ways can you travel from station 1 to station 3 without passing through the same station twice?
   c. How many different ways can you travel directly from station 1 to station 3 with no stops?
   d. How many different ways can you travel from station 1 to station 3 with exactly one stop?
   e. How many different ways can you travel from station 1 to station 3 with exactly two stops? Allow for stops at repeated stations.

5. Suppose the matrix R represents a railroad map between cities 1, 2, 3, 4, and 5.

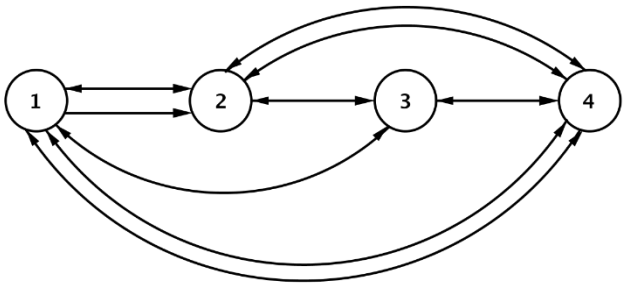
$$R = \begin{bmatrix} 0 & 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 & 2 \\ 1 & 1 & 0 & 0 & 2 \\ 1 & 1 & 3 & 0 & 0 \end{bmatrix}$$

   a. How many different ways can you travel from City 1 to City 3 with exactly one connection?
   b. How many different ways can you travel from City 1 to City 5 with exactly one connection?
   c. How many different ways can you travel from City 2 to City 5 with exactly one connection?

6. Let $B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$ represent the bus routes between 3 cities.
   a. Draw an example of a network diagram represented by this matrix.
   b. How many routes are there between City 1 and City 2 with one stop in between?
   c. How many routes are there between City 2 and City 2 with one stop in between?
   d. How many routes are there between City 3 and City 2 with one stop in between?
   e. What is the relationship between your answers to parts (b)-(d)? Formulate a conjecture.

II. Problem Solving

1. Consider the airline flight routes between Cities 1, 2, 3, and 4, as shown.



   a. Create a matrix F to represent the flight map between Cities 1, 2, 3, and 4.

b. How many different routes can you take from City 1 to City 4 with no stops?

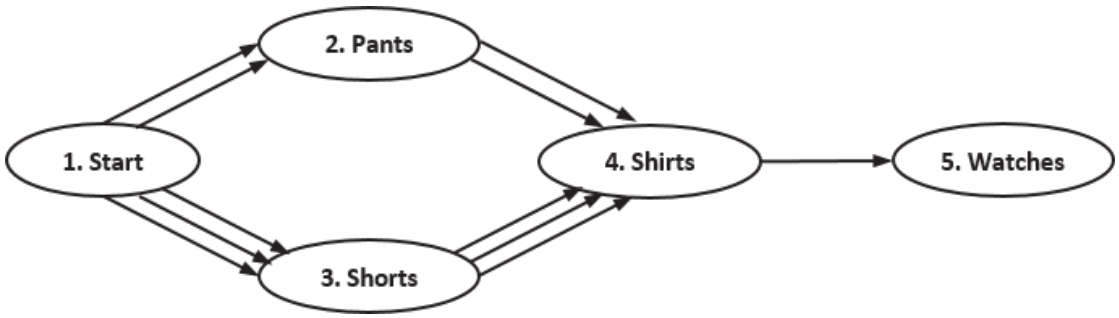
c. How many different routes can you take from City 1 to City 4 with exactly one stop?

d. How many different routes can you take from City 3 to City 4 with exactly one stop?

e. How many different routes can you take from City 1 to City 4 with exactly two stops? Allow for routes that include repeated cities.

f. How many different routes can you take from City 2 to City 4 with exactly two stops? Allow for routes that include repeated cities.

2. Consider the following directed graph representing the number of ways Trenton can get dressed in the morning (only visible options are shown):



a. What reasons could there be for there to be three choices for shirts after "traveling" to shorts but only two after traveling to pants?

b. What could the order of the vertices mean in this situation?

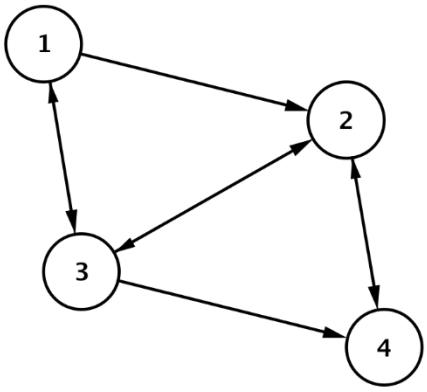
c. Write a matrix A representing this directed graph.

d. Delete any rows of zeros in matrix A, and write the new matrix as matrix B. Does deleting this row change the meaning of any of the entries of B? If you had deleted the first column, would the meaning of the entries change? Explain.

e. Calculate $b_{1,2} \cdot b_{2,4} \cdot b_{4,5}$. What does this product represent?

f. How many different outfits can Trenton wear assuming he always wears a watch?

3. Faced with competition from rival companies, you have been tasked with considering the option of building a toll road going directly from City 1 to City 4. Once built, the road will provide income in the form of tolls and also enable the implementation of a non-stop bus route to and from City 1 and City 4.



Analysts have provided you with the following information (values are in millions of dollars):

	Start-up costs (expressed as profit)	Projected minimum profit per year	Projected maximum profit per year
Road	−\$63	\$65	\$100
New bus route	−\$5	\$0.75	\$1.25

a. Express this information in a matrix P.

b. What are the dimensions of the matrix?

c. Evaluate $p_{1,1} + p_{1,2}$. What does this sum represent?

d. Solve $p_{1,1} + t \cdot p_{1,2} = 0$ for t. What does the solution represent?

e. Solve $p_{1,1} + t \cdot p_{1,3} = 0$ for t. What does the solution represent?

f. Summarize your results to part (d) and (e).

g. Evaluate $p_{2,1} + p_{2,3}$. What does this sum represent?

h. Solve $p_{2,1} + t(p_{2,3}) = 0$ for t. What does the solution represent?

i. Make your recommendation. Should the company invest in building the toll road or not? If they build the road, should they also put in a new bus route? Explain your answer.