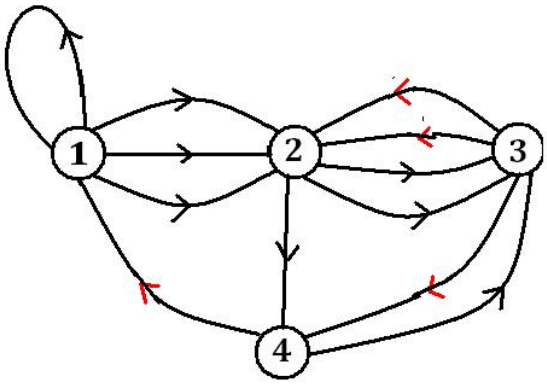
4.2: The Matrix

Matrices and Networks, Part 2

Welcome back, Traveler!

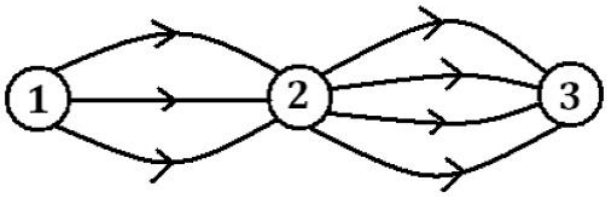
Now that you know a little Graph Theory, let's see if it helps you on your mission to rescue Zion. Here is the route map again:



I. Introduction to Matrices

Keep in mind that we are trying to find out how many possible routes exist between Mega City (City 1) and Zion (City 3).

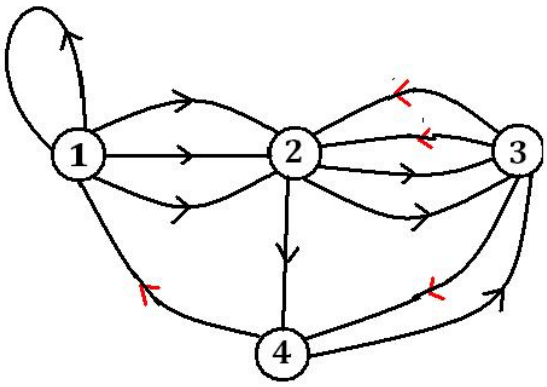
Let's try a simpler problem! Imagine this network represents the travel options between three cities.



1. How many routes are there *now* between City 1 and City 3?

2. Niobe claims that the correct answer is 7 routes. "There are 3 routes between City 1 and City 2. There are four routes between City 2 and City 3. So, all you have to do is add the two numbers together." Explain the error in Niobe's reasoning.

3. Make a conjecture that predicts the number of routes between two cities that have no direct connection based on the number of connections (say m and n) they have with a third city.



Counting the possible routes is still confusing! Let's try to organize the information.

4. List all the direct routes between the cities in the table below:

Destination Cities

Cities of Origin	1	2	3	4
1	1			
2				
3				
4				

Let's call this table P, where P stands for "paths," and use the notion $P_{i,j}$ for the entry in the i^{th} row and j^{th} column of the table, just as people usually do. So:

- $P_{1,2} = 3$, because there are three direct paths from city 1 to city 2.
- $P_{3,4} = 1$, because there is one direct route from city 3 to city 4

and so on. Got it?

5. Find the value of:
    a. $P_{4,1} =$
    b. $P_{1,4} =$
    c. $P_{2,2} =$

6. Given that it is understood that the rows and columns represent cities, we don't really need to write down the borders and labels of the table. Copy the 16 numbers from the center of the table – arranged in rows and columns, but no table features.
    a. Add parentheses around the block of numbers – yes, big ones!
    b. Add a **P =** in front of the parentheses.
    c. Congratulate yourself on creating a matrix.

TERMINOLOGY:

A **matrix** is an array of numbers organized into m rows and n columns. The **entry** in the first row and first column is referred to as $a_{1,1}$. In general, the entry in the i^{th} row and j^{th} column would be denoted $a_{i,j,}$.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 11 & -1 \\ 4 & 3 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

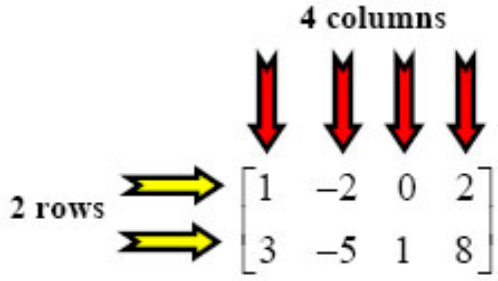
$$\mathbf{C} = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 7 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} -1 & 0 & 4 \\ 3 & 1 & 1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} 1 & -1 \\ -2 & 0 \\ 1 & -2 \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} -5 & 2 \\ -1 & 3 \\ 4 & -4 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 \\ 3 & -1 & -2 \\ 0 & 2 & -1 \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\mathbf{x} = [\,2 \;\; -1 \;\; 1 \;\; -2\,] \qquad \mathbf{y} = [\,-3 \;\; 2 \;\; 1 \;\; -6\,]$$

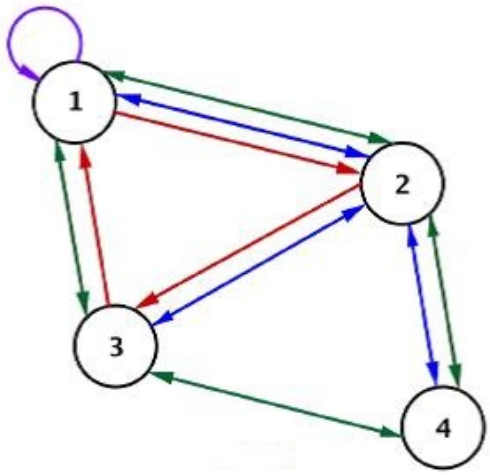
A matrix containing m rows and n columns has **dimensions** $m \times n$.



Dimensions: (2 x 4)

7. What are the dimensions of the P (the Mega City to Zion route matrix you made in question 3)?

8. An airline offers more routes connecting the four cities as shown below. Use the network diagram below to represent the number of direct routes between the four cities in a matrix R.



9. What is the value of $R_{2,3}$? What does it represent in this situation?

10. What is the value of $R_{2,3} \cdot R_{3,1}$, and what does it represent in this situation?

11. Write an expression for the total number of one-stop routes from City 4 and City 1, and determine the number of routes stopping in one city.

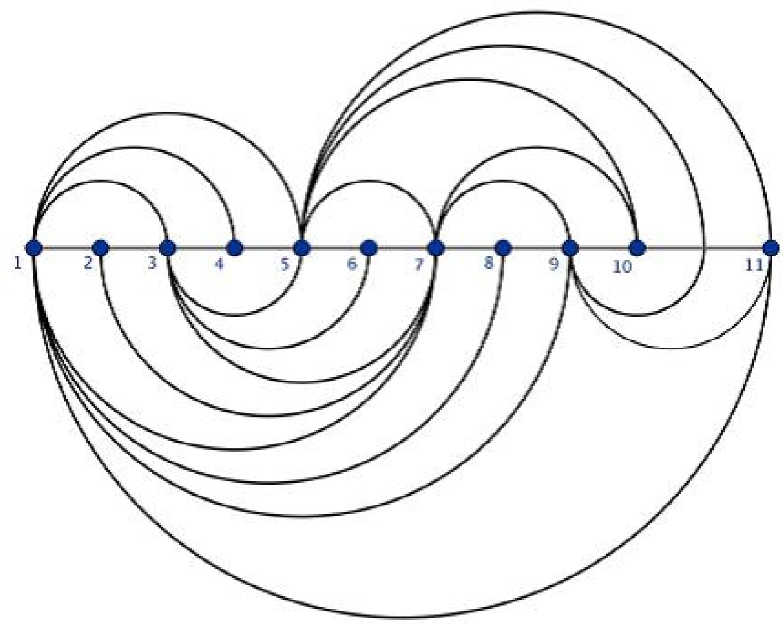
12. Do you notice any patterns in the expression for the total number of one-stop routes from City 4 and City 1?

13. Create a network diagram for the matrices shown below. Each matrix represents the number of transportation routes that connect four cities. The rows are the cities you travel from, and the columns are the cities you travel to.

a. $R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

b. $R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

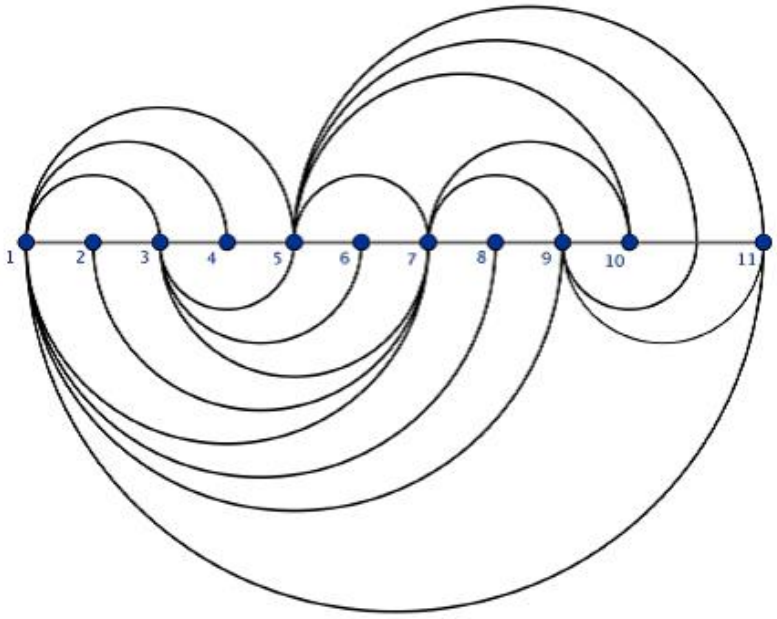
Here is a type of network diagram called an arc diagram.



14. Suppose the points represent eleven students in your mathematics class, numbered 1 through 11. Suppose the arcs above and below the line of vertices 1–11 are the people who are friends on a social network.

    a. Complete the matrix that shows which students are friends with each other on this social network. The first row has been completed for you.

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$



b. Number 1 is not friends with number 10. How many ways could number 1 get a message to 10 by only going through one other friend?

c. Who has the most friends in this network? Explain how you know.

d. Is everyone in this network connected at least as a friend of a friend? Explain how you know.

e. What is entry $A_{2,3}$? Explain its meaning in this context.