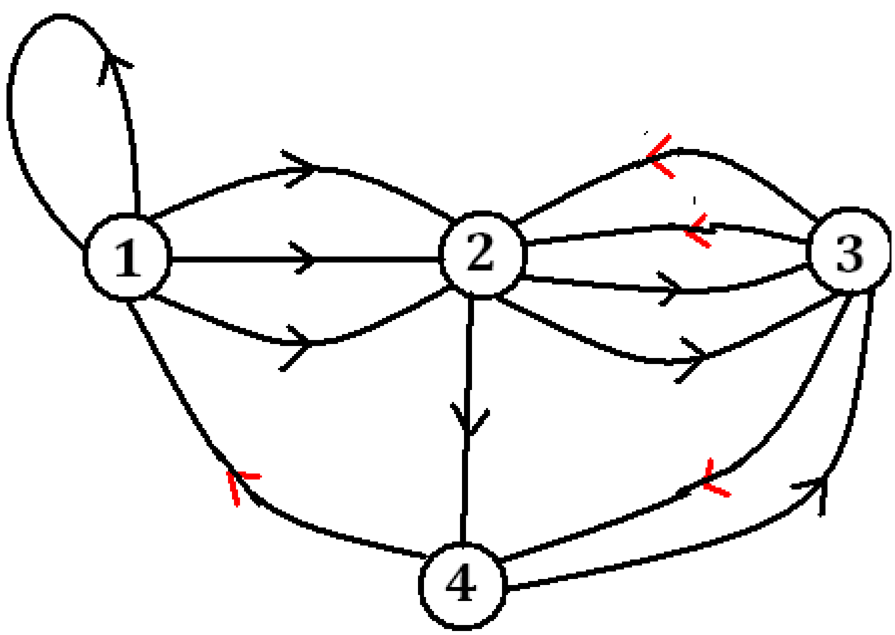
4.1: Networks

Graph Theory and Networks

Hello Traveler! Welcome to Mega City!

As you may know, we received an important package that must be delivered to Zion. The fate of that human city depends on the successful completion of your mission to deliver the package. Unfortunately, there is no direct route from Mega City to Zion.

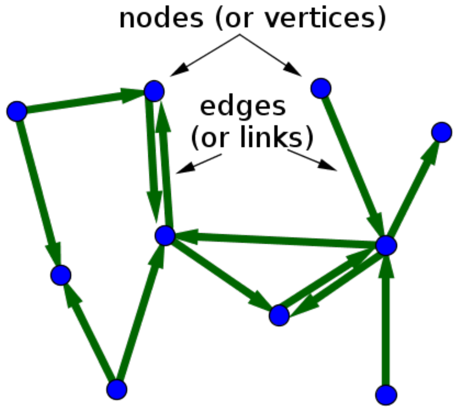
> *There are, however, three routes you can choose to travel from Mega City to Westview, and there are four from there to Zion… and a few more, as detailed in the "route map" below.*



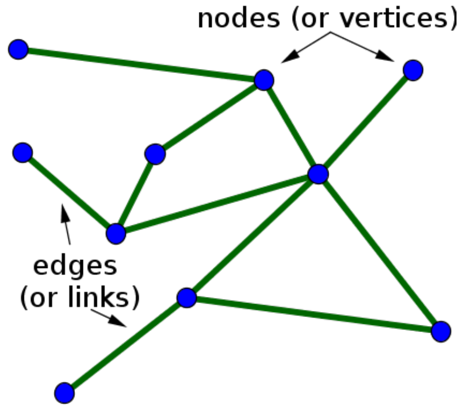
Task 1: How many different routes can one take to travel from city 1 to city 3?

I. Introduction to Networks

A **network** is a set of objects (called **nodes** or vertices) that are connected together. The connections between the nodes are called **edges** or links. In mathematics, networks are often referred to as graphs.

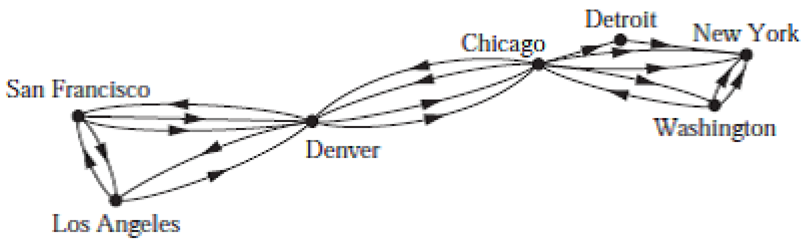


Directed Graph



Undirected Graph

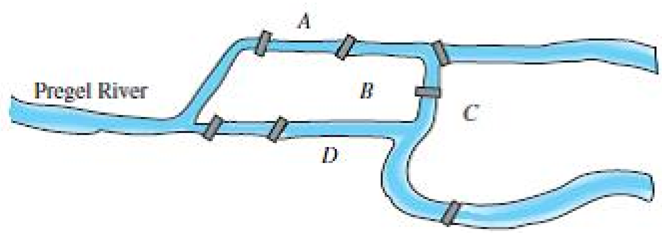
If the edges in a network are directed, i.e., pointing in only one direction, the network is called a directed network (or a directed graph). If all edges are bidirectional, or undirected, the network is an undirected network or undirected graph.



1. Refer to the graph above:
    a. Is this a directed or undirected graph? Explain your reasoning.
    b. How many nodes are there?
    c. Which edges are bidirectional?
    d. Open-ended: Give a brief description of a real-life scenario that could be modeled by this graph.

II. Graph Theory

The subject of graph theory began in the year 1736 when the great mathematician Leonhard Euler published a paper giving the solution to the following puzzle: The town of Königsberg in Prussia (now Kaliningrad in Russia) was built at a point where two branches of the Pregel River came together. It consisted of an island and some land along the river banks. These were connected by seven bridges as shown in the figure below.

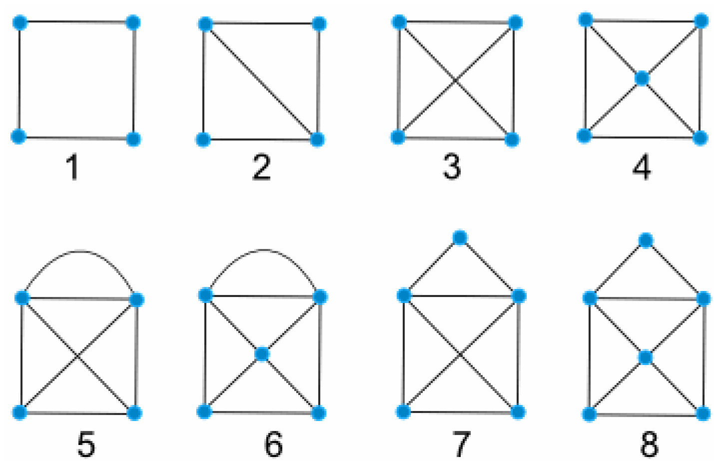


2. Is it possible for a person to take a walk around town, starting and ending at the same location and crossing each of the seven bridges exactly once?[1] Justify your response.

3. Create an undirected network of Euler's Königsberg Bridge problem.

[1] In his original paper, Euler did not require the walk to start and end at the same point. The analysis of the problem is simplified, however, by adding this condition. Later in the section, we discuss walks that start and end at different points.

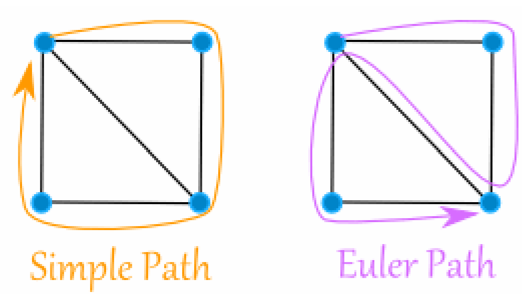
4. Let's take a step back and try some simpler shapes. Try to draw each figure, but never go over any line more than once, and don't remove your pencil from the paper. Record your results in the table below:



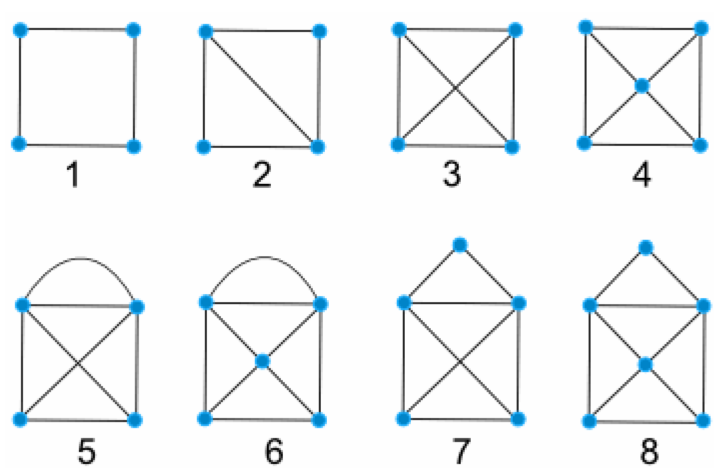
Graph	Success?
1	Yes
2	
3	
4	
5	
6	
7	
8	

Digging Deeper: Whether a successful Euler path exists has something to do with the number of edges that lead to (or connect with) a vertex, which is called the **degree** of the vertex.

And, yes the precise name of a route around a graph that visits *every edge once* is called an **Euler path.** A route around a graph that visits *every vertex once* is called a **simple path.**



5. Count the number of odd- and even-numbered vertices in the 8 graphs below and record your data in the table below. Also include the total number of vertices to help check your accuracy.



Graph	Euler Path?	Total Vertices	# of Odd Degree Vertices	# of Even Degree Vertices
1	Yes			
2				
3				
4				
5				
6				
7				
8				

6. Make a conjecture about Euler paths and the degree of the vertices.

7. Return to Königsberg: OK, imagining that the edges are bridges, and you cross them once only, you have solved the Königsberg puzzle.

   Is it possible for a person to take a walk around town, starting and ending at the same location and crossing each of the seven bridges exactly once? Justify your response.

8. Which of the following graphs have Euler paths?