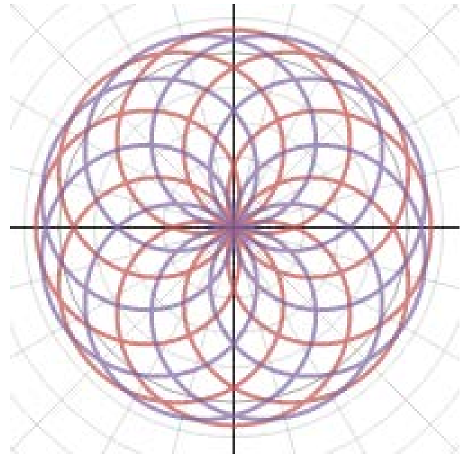
2.6 Roses, Rings and Hearts… oh my!

Polar Curves



Many companies, brands, or organizations have a symbol or logo that instantly identifies them to the world. Graphic designers and advertising firms often create designs like the ones shown below to represent their client's image without words.

The symbol shown above can be represented by a **polar equation**. In this activity, you will investigate graphs such as these and their equations in both polar and rectangular form.

Driving Question: How can we use polar graphs to create a company logo?

This activity takes advantage of [Desmos], a free online graphing utility. If you have not done so already go to the [Desmos] web site and register for a free account. It is possible to complete this activity with some graphing calculators.

1. Enter the polar equation $r = 3$. Desmos automatically recognizes this as a polar equation.

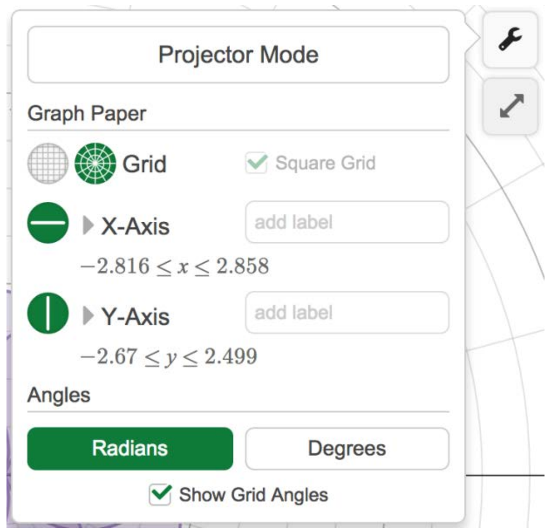
   a. Describe the characteristics of the graph of the equation

   b. This is a single variable equation (r), similar in some ways to $x = 3$ and $y = 3$. Why is the graph of $r = 3$ **not** a straight line?

   c. Write the rectangular equation for the graph.

   d. Write the rectangular equation for a polar equation in the form $r = a$.

   e. Set the grid mode to Polar: Click on the Graph Settings button (with a wrench) in the upper right corner. Click on the Polar Grid image and de-select the x- and y- axes.



2. Consider the polar equation $\theta = \frac{\pi}{3}$.

a. Try graphing the equation in Desmos. (Desmos does not allow you to enter polar equations in this form – much like some calculators only graphing rectangular equations in $y =$ form. Rewrite the equation in r = form)

b. Describe the characteristics of the graph of the equation.

c. Write the rectangular equation for the graph.

d. Delete all equations.

3. Enter the following polar equations on separate lines: $r = 3\theta$, $r = 4\theta$, and $r = \frac{1}{2}\theta$.

Consider the polar equation of the form $r = a\,\theta$.

a. Compare and contrast the characteristics of the graphs of $r = 3\theta$, $r = 4\theta$, and $r = \frac{1}{2}\theta$.

b. Write a general statement describing the effect of the coefficient a on the graph of a polar equation in the form $r = a\,\theta$.

4. Enter the following polar equations on separate lines: $r = 3\cos(\theta)$, $r = -4\cos(\theta)$, and $r = \frac{1}{2}\cos(\theta)$. Consider the polar equation of the form $r = a\,cos(\theta)$.

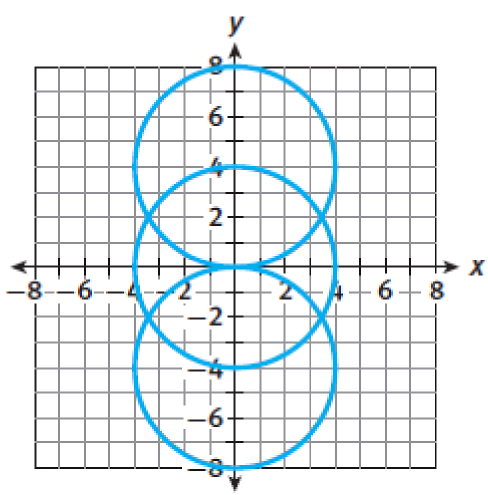
a. Compare and contrast the characteristics of the graphs of $r = 3\cos(\theta)$, $r = -4\cos(\theta)$, and $r = \frac{1}{2}\cos(\theta)$.

b. Describe, without graphing, the characteristics of $r = 2\cos(\theta)$. Be sure to include a description of the size and location of the graph.

c. Confirm your description in Item 3b by graphing. Sketch all four polar equations on the graph below.

d. Write a general statement describing the effect of the coefficient a on the graph of a polar equation in the form $r = a\cos(\theta)$.

e. Describe the symmetry of and effect of the coefficient a on the graph of a polar equation in the form $r = a\,\mathbf{sin}(\theta)$.

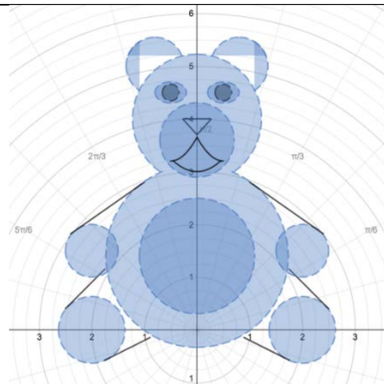
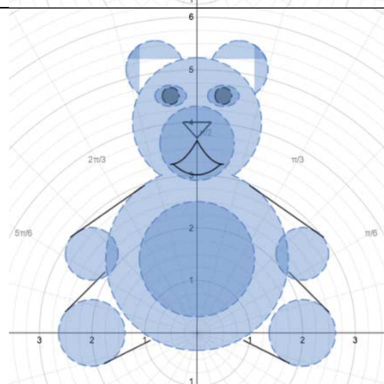
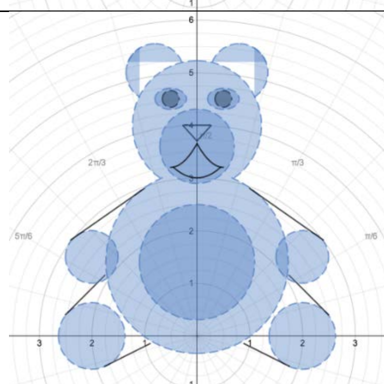
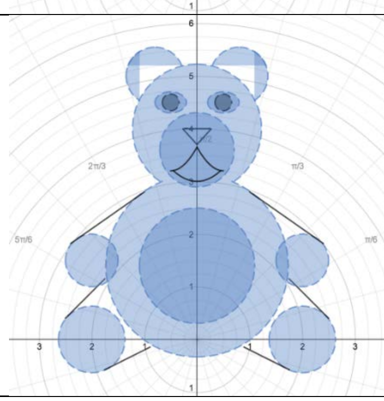


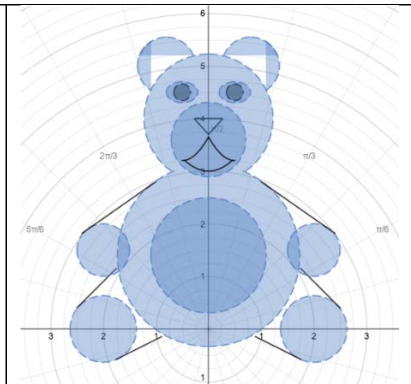
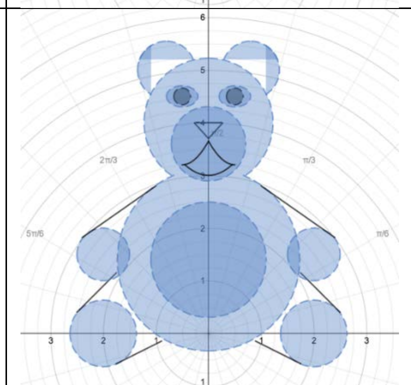
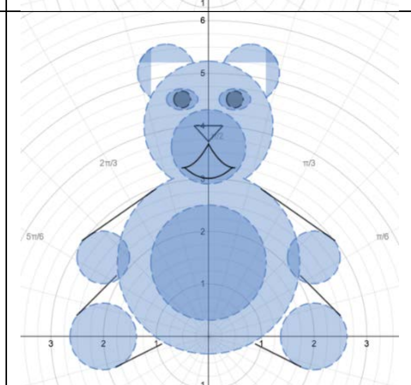
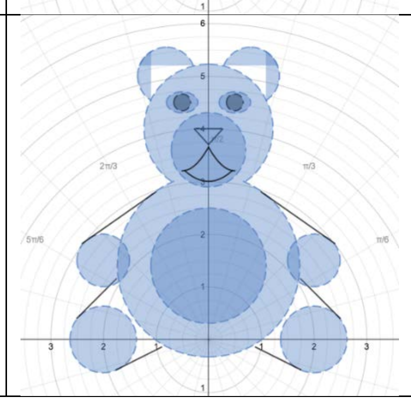
5. Consider the design above.

a. Write the polar equation for each of the circles shown in the design.

b. Write the rectangular equation for each of the circles shown in the design.

6. Enter each of the polar equations from the table below, graph the curve, and complete the table.

Equation	Graph	Number of Petals	Length of each Petal
$r = 2\sin 4\theta$			
$r = 2\sin 5\theta$			
$r = -3\sin 2\theta$			
$r = -4\sin 3\theta$			

$r = 2\cos 4\theta$			
$r = 2\cos 5\theta$			
$r = -3\cos 2\theta$			
$r = -4\cos 3\theta$			

The graph of a polar equation in the form $r = a\cos(n\theta)$ or $r = a\sin(n\theta)$ is called a **Rose curve.**

7. Look for patterns in the table you completed in Question 6 to determine the following.

a. How does changing the value of a influence the characteristics of the graph of a rose curve?

b. How does changing the value of n influence the characteristics of the graph of a rose curve?

c. Delete all the equations. Enter the parametric equation $r = a\cos(n\theta)$. Add ALL sliders for parameters a and n.

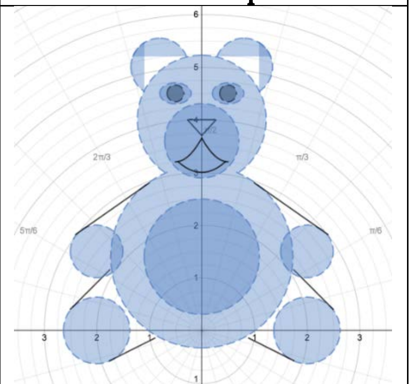
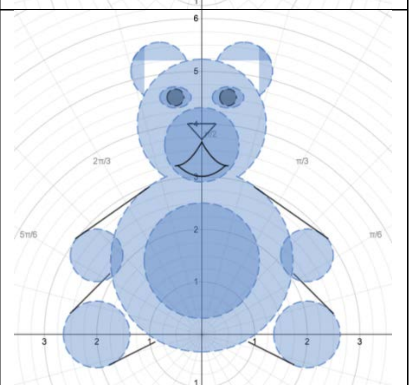
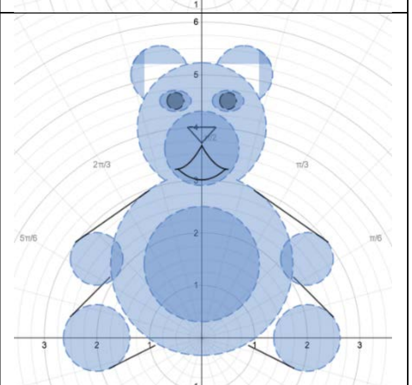
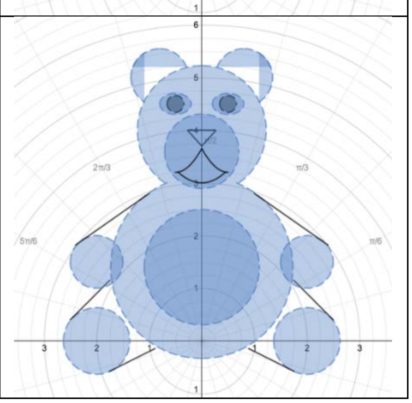
d. Experiment with the sliders to confirm/edit your responses to parts **a** and **b**. Find an interesting new design and sketch it below. Write the polar equation next to it.

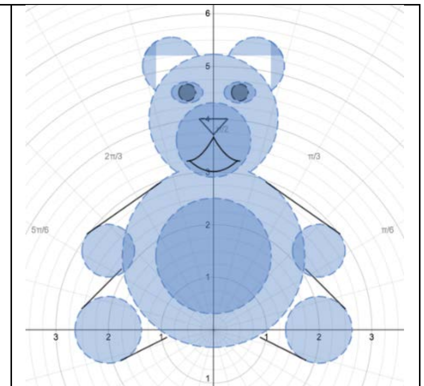
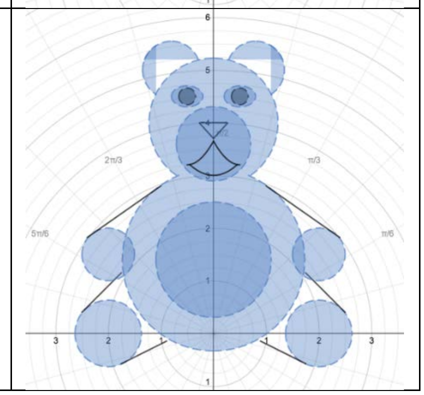
e. Enter the parametric equation $r = a\sin(n\theta)$.

f. Describe the differences in the symmetry of the graph of $r = a\cos(n\theta)$ versus $r = a\sin(n\theta)$. (You can hide either equation by clicking on its graph icon.

The graph of the polar equation in the form $r = a \pm b\sin\theta$ or $r = a \pm b\cos\theta$ with $a = b$ is called a **cardioid.** If $a \neq b$ the graph is called a **limaçon.**

8. Consider the graph of a polar equation in the form $r = \mathrm{a} \pm b \sin\,\theta$ or $r = a \pm b \cos\,\theta$. Select values of a and b that meet each condition. Graph several equations and then record a sketch labeling key points on the graph.

<table>
<tr><th>Equation</th><th>Condition</th><th>Your Equation</th><th>Your Graph</th></tr>
<tr><td>$r = a \pm b \sin\theta$</td><td rowspan="2">$a = b$</td><td></td><td></td></tr>
<tr><td>$r = a \pm b \cos\theta$</td><td>Type equation here.</td><td></td></tr>
<tr><td>$r = a \pm b \sin\theta$</td><td rowspan="2">$a > b$</td><td>Type equation here.</td><td></td></tr>
<tr><td>$r = a \pm b \cos\theta$</td><td>Type equation here.</td><td></td></tr>
</table>

$r = a \pm b\sin\theta$		Type equation here.	
$r = a \pm b\cos\theta$	$a < b$	Type equation here.	

You have studied the standard rectangular form of the equations of conic sections. When one focus of a conic is located at the pole, conic sections can be written in a standard polar form.

9. Conic sections having one focus located at the pole can be expressed in the form $r = \frac{b}{a \pm c \cdot cos\theta}$ or $r = \frac{b}{a \pm c \cdot sin\theta}$

a. The values of a and c determine whether the graph of the conic section will be a parabola, ellipse, or hyperbola. Make a conjecture about the values of a and c that will determine whether the conic section graphed will be a parabola, ellipse, or hyperbola. Defend your conjecture with examples.

b. What effect does the parameter b have on the graph of the conic section?

c. What effect does changing the sine to cosine have on the graph of the conic section?

10. Determine whether the equation gives the graph of a parabola, ellipse, or hyperbola, and describe the symmetry of the graph. Explain.

a. $r = \frac{4}{3-2\cdot sin\theta}$

b. $r = \frac{4}{2-3\cdot cos\theta}$

c. $r = \frac{4}{3+3sin\theta}$

11. The **lemniscate** below was graphed in Desmos using the sine function. Look up the formula and re-graph using the cosine function. Replace the image below with your cosine graph. (Desmos won't like it at first; see if you can find an equivalent equation, that is oriented like a standard infinity symbol.