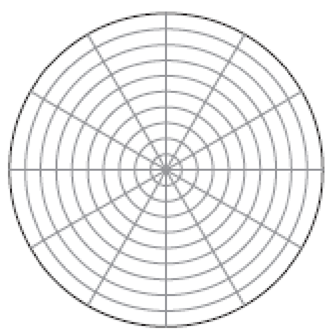
2.5: Air Traffic Controller

Polar Graphs

Air traffic controllers play an important role in keeping the aviation skies safe. Their job is to monitor air space around an airport, typically within a 50-mile radius, in order to manage the traffic of airplanes flying in this air space. They monitor air traffic by keeping track of the location and movement of planes on a radar screen.

Photo Credit: Eurosys [http://www.eurosys.com/air-traffic-control-siemens/]

Driving Question: How can we use polar graphs to increase air safety?



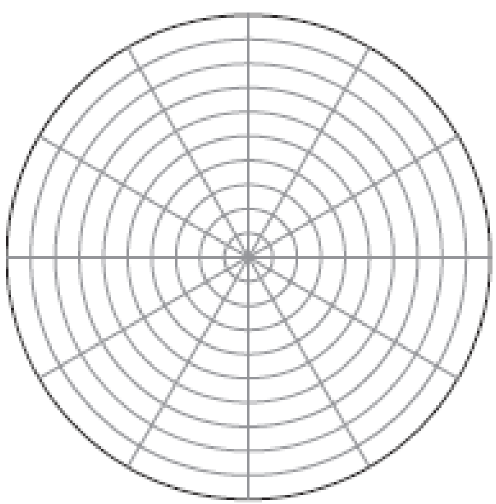
Air Traffic Controller
Radar Screen

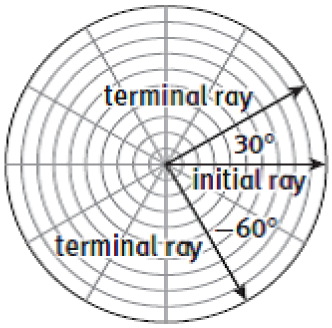
The grid on the radar screen is called a **polar grid**. The grid is made up of concentric circles and rays from the common center of these circles. Suppose that the common center of these circles is point O. Point O is called the **pole** of the polar grid. Point O gives the location of the air traffic control tower, which is where the air traffic controllers work. Thus, all air traffic is referenced with respect to pole O, the position of the control tower.

1. In the polar grid on the radar screen, each concentric circle has a radius equal to a multiple of 5 miles, with the smallest circle having a 5-mile radius and the largest circle having a 50-mile radius. The rays from the center of the polar grid form angles that are multiples of 30°. Describe a method that can be used to verify that the angles formed by the rays are multiples of 30°.

An air traffic controller must be familiar with the polar grid. When airplanes enter the control tower's air space, they are identified by radar and their location is displayed on the radar screen by a brightened point on the polar grid. This point is a **screen pixel**. Air traffic controllers often identify the screen pixel by referring to it as "the aircraft being tracked."

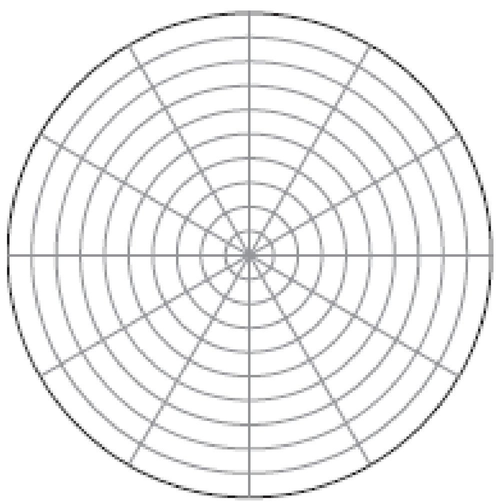
2. An airplane is located 30 miles from the air traffic control tower. Indicate on the polar grid, and describe in words, the set of points (pixels) where the airplane could be located.





As aircraft move through the air space, radar tracks their movement by periodically updating their locations. In addition to a distance to locate an airplane an angle is also needed. The **initial ray** for this angle is the ray from the pole *O* along the positive *x*-axis, indicated by the dark ray in the polar grid shown at left. The **terminal ray** for this angle is the ray from the pole passing through the point locating the airplane's position on the radar screen. The angle is determined by rotating the initial ray into the terminal ray. If this rotation is counterclockwise, the angle has a positive measure. If it is clockwise, the angle has a negative measure.

To locate an airplane on the radar screen, an ordered pair is used, a distance from the pole *O* and an angle from the polar axis. The **polar coordinates** for a point on the polar grid are given as (r, θ), where *r* gives the distance from the pole *O* and θ gives the angle from the polar axis. A point that is 50 miles from *O*, at an angle of 90° from the polar axis could be written as the ordered pair (50, 90°).



3. On the grid above, plot and label the following points. Then, write the polar pairs (r, θ) for the points that were described.

a. Point *A*, 20 miles from *O*, at an angle of 60° from the polar axis.

b. Point *B*, 20 miles from *O*, at an angle of -120° from the polar axis.

c. Point *C*, 15 miles from *O*, at an angle of 150° from the polar axis.

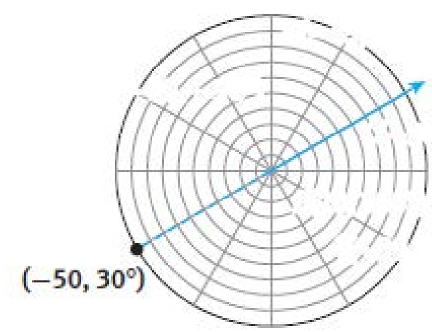
4. Consider the points (40, 240°) and (40, -120°).

a. Describe, in words, what occurs when you plot these points.

b. A rectangular *xy*-coordinate grid uses ordered pairs of the form (x, y) to indicate points in the plane. Do the rectangular pairs of the form (x, y) have the same property that you observed for the polar pairs (40, 240°) and (40, -120°)? Justify your explanation.

c. Name some other polar pairs that identify the points (40, 240°) and (40, -120°). Explain how such other polar pairs can be constructed.

The r-values in the examples in Items 1–3 were given as positive values marking the distance of the point from the pole. In general, the r-value in a polar ordered pair (r, θ) is a directed (positive or negative) distance from the pole. The value r can be given as a negative number. If θ is a directed angle, and if $r > 0$, the location of the point on the polar grid is found by moving along the terminal ray, making the angle θ. If $r < 0$, the location of the point on the polar grid is found by extending the terminal ray in the opposite (negative) direction, and moving along this opposite extension of the ray, a distance of $| r /$units from pole O. The point (-50, 30) is shown on the graph below.



5. Plot and label the following polar ordered pairs on the polar grid shown above.

a. Point A: (-30, 120°)

b. Point B: (-45, 270°)

c. Point C: (-30, -210°)

d. Point D: (-25, -90°)

e. Locate the point given by the polar pair for Point E: (20, 300°) on the polar grid above. Next, list three different polar ordered pairs (r, θ) which describe the same point on this grid.

6. Points on the polar grid can have multiple polar pairs denoting their location.

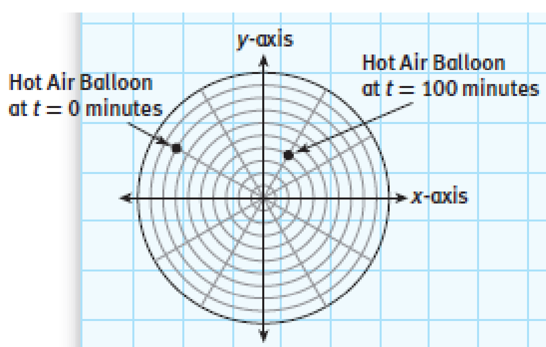
a. Suppose $|r_1| = |r_2|$. Must polar pairs $(r1, \theta_1)$ and (r_2, θ_1) denote the same point on the polar grid? Explain why or why not.

b. Suppose $r_1 = -r_2$, and the polar pairs (r_1, θ_1) and (r_2, θ_2) denote the same point on the polar grid. What can you conclude about the angles θ_1 and θ_2?

c. Suppose the polar pairs (r, θ_1) and (r, θ_2) denote the same point on the polar grid. What can you conclude about the angles θ_1 and θ_2?

Although air traffic controllers work with radar screens that require using polar coordinates to locate aircraft, it is important for controllers to translate a location, shown in polar coordinates (r, θ) on their radar screens, into rectangular coordinates. The rectangular coordinates will be referenced to the standard directions of north-south (y-axis directions) and east-west (x-axis directions).

One afternoon, a hot air balloon suddenly appeared as a blip on the air traffic controller's radar screen. To air traffic controllers, hot air balloons can mean trouble. Hot air balloons are prohibited from flying within a 20-mile radius of the airport. (The circles in the grid below are at 5-mile intervals.)



7. The polar grid shows the blip for the hot air balloon at time $t = 0$ minutes and at a time 100 minutes later.

a. Express the two locations of the hot air balloon in polar coordinates.

b. Is the hot air balloon in violation of the airport air space at either of these locations? Explain your reasoning.

The air traffic controller must warn the hot air balloon pilot of its possible encroachment of the airport air space. A police car is sent to establish visual contact with the pilot using the police car's warning lights. The police officers will then use their amplified speakers to convey the warning to the balloonist. Rectangular north-south-east-west directions, referenced from the control tower with the origin placed at the pole, can be used to convey the exact location of the hot air balloon to the police.

Notice that the axes from the rectangular *xy*-coordinate system, with the origin placed at the pole and the positive *x*-axis along the polar axis, has been superimposed on the polar grid. Right triangle trigonometry can be used to determine the rectangular coordinates for a polar point.

EXAMPLE 1

Determine the rectangular coordinates for (40, 30°).

Step 1: *Find the x coordinate.*

$$\cos 30° = \frac{x}{40}$$

$$x = 40 \cos 30°$$

$$x = 20\sqrt{3} \approx 34.64$$

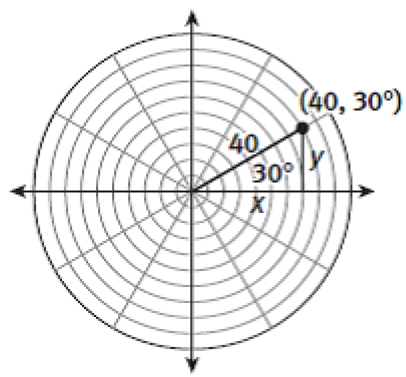
Step 2: *Find the y coordinate.*

$$\sin 30° = \frac{y}{40}$$

$$y = 40 \sin 30°$$

$$y = 20$$

Solution: $(x, y) = (20\sqrt{3}, 20)$



8. Your Turn

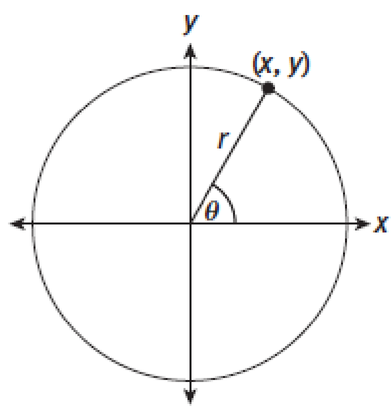
a. Use right triangle trigonometry to determine the rectangular coordinates for the hot air balloon's location when $t = 100$ minutes. Show your work and explain your reasoning.

b. How many miles east and how many miles north of the air traffic control tower is the hot air balloon at $t = 100$ minutes?

c. What are the rectangular coordinates for the location of the hot air balloon at $t = 0$ minutes?

Air traffic controllers must be prepared for emergencies that require working with area police and fire services. Since fire and police agencies work with rectangular coordinates, generalized expressions translating (r, θ) coordinates into (x, y) coordinates are needed.

9. Given a point described by the polar pair (r, θ), use the diagram below and trigonometry to express the rectangular coordinates of x and y in terms of r and θ. Show work to support your answer.



10. Apply the results in Item 6 to write the rectangular coordinates (x, y) for the following polar coordinates (r, θ).

a. The rectangular coordinate pair for (20, 150°) is:

b. The rectangular coordinate pair for (12, 225°) is:

c. The rectangular coordinate pair for (10, –30°) is:

d. The rectangular coordinate pair for (-30, -315°) is:

e. The rectangular coordinate pair for (17, 128°) is:

By the time the police are able to catch up to the hot air balloon, it is located 3 miles east and $3\sqrt{3}$ miles south of the control tower. The balloon is on the ground, but not yet secured. The police report this location to the control tower and the air traffic controller records it in rectangular form as $(3, -3\sqrt{3})$. Because the balloon is on the ground, radar is unable to detect the balloon. Until the balloon is secured, it continues to pose a danger to air traffic in the area, especially if a gust of wind should suddenly send it airborne once again.

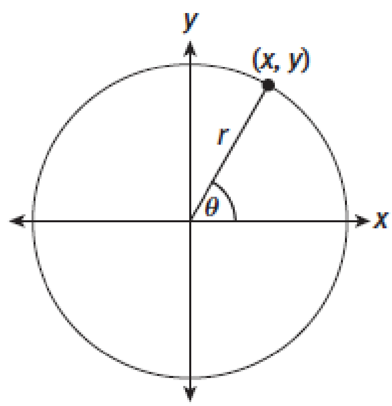
11. The air traffic controller must mark the balloon's location on the radar screen.

> **a.** Based on the rectangular coordinates for the hot air balloon, determine if the balloon's position violated the airport's 20-mile limit by finding how far the balloon is located from the control tower. Show your work.
>
> **b.** What angle θ could be used to locate the hot air balloon on the polar grid? Show your work.
>
> **c.** Give a list of four polar coordinate pairs (r, θ) that could be used to locate the hot air balloon's position.

12. Suppose that before the hot air balloon could be secured, a strong gust of wind pushed the balloon to a new location. When the balloon was finally secured, its new position in relation to the control tower was $(-2\sqrt{3} , -2)$.

> **a.** How far from the control tower is the hot air balloon at this new position? Show your work.
>
> **b.** Give two possible angles for θ that could be used for the hot air balloon's polar coordinates (r, θ). Show your work.

13. Given a point described by the rectangular pair (x, y), use the diagram below, and trigonometry, to express the polar coordinates of r and θ in terms of x and y. Show your work.



14. An approaching aircraft is 9 miles west and $9\sqrt{3}$ miles north of the air traffic control tower.

a. Determine the rectangular coordinates of the aircraft .

b. Use your results in Item 13 to determine the polar coordinates for this aircraft .

c. Use your calculator to compute the angle for the aircraft at location (-9, $9\sqrt{3}$) and explain why the calculator gives the angle for the aircraft as located at (9, $-9\sqrt{3}$).

15. Convert the rectangular coordinates to polar points.

a. (-12, -9) **b.** (6, - 8) **c.** (-12, 5)

Pilots have a radar screen in the cockpit of their airplane. When landing in overcast weather conditions, the radar screen becomes their "eyes," guiding them to the runway for landing their aircraft . This radar screen is also a polar grid, with the pole on the screen giving the location of the Instrument Landing System (ILS) transmitting tower and the polar axis denoting due east from the ILS. The brightened point on the screen shows the location of the aircraft relative to the ILS transmitting system.

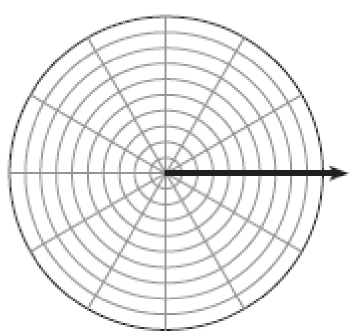
Typically, the ILS transmitting system is positioned at the ends of a runway. This positioning shows the pilot exactly where the beginning and end of the runway is located. From the ILS transmission, the pilot uses the cockpit radar screen to maneuver the airplane toward the pole, which locates the beginning of the runway.

Polar curves of the form $r = f(\theta)$ can be used to describe landing approaches for aircraft pilots. For the polar grids in these questions, the scale along the polar axis is one.

16. The ILS signal indicates that the landing approach is toward the south. The scope on the airplane uses concentric circles with radii in steps of 1 mile from the ILS runway signal.

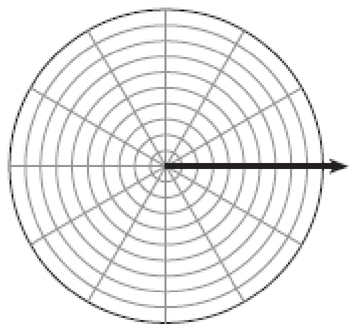
a. Use the polar grid below to sketch the landing approach that is described by the polar function $r = 6 \cdot \cos(\theta)$ for $0° \leq \theta \leq 90°$.

Indicate the location of the plane when $\theta = 0°$, $30°$, $60°$, and $90°$.



b. Describe the type of landing approach that is sketched in Part a.

c. Your graph represents the landing approach for aircraft . However, the graph of this landing approach is only part of the complete graph of a polar function of the form $r = f(\theta)$. Use the polar grid shown below to give a complete graph of $r = 6 \cdot \cos(\theta)$ in the *xy*-coordinate plane. Indicate the interval of values for θ that gives the complete graph.



The polar function $r = 6 \cdot \cos(\theta)$ can be transformed into rectangular variables x and y.

Since $r = \sqrt{x^2 + y^2}$ and $\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$, using substitution, $r = 6 \cdot \cos(\theta)$ can be rewritten as $\sqrt{x^2 + y^2} = 6(\frac{x}{\sqrt{x^2+y^2}})$ and simplified to $x^2 + y^2 = 6x$.

d. Does this transformation confirm your answer to Part b? Explain.

17. Your Turn

a. Sketch the complete graph of $r = 4 \cdot \sin(\theta)$

b. Transform the polar function $r = 4 \cdot \sin(\theta)$ into rectangular variables x and y.