2.4 Hyperbolic Navigation

Practice Tasks

I. Concepts and Procedures

1. A hyperbola is the set of all points in the plane for which the _____ of the distances from two fixed points F_1 and F_2 are called the _____ of the hyperbola.

2. The graph of the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with $a > 0, b > 0$ is a hyperbola with vertices (_____, _____) and (_____, _____) and foci ($\pm$c , 0) where $c =$ _____ . So the graph of (_____, _____) is an hyperbola with vertices (_____, _____) and (____, ____) and foci (_____, _____) and (_____, _____).

3. Write a clever memory device to differentiate between the equation of an ellipse and a hyperbola.

4. Find the vertices, foci, and asymptotes of the hyperbola, and sketch the graph:

   a. $\frac{x^2}{4} - \frac{y^2}{16} = 1$

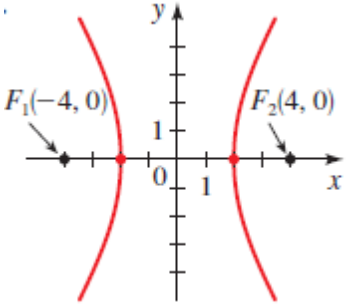
   b. $y^2 - \frac{x^2}{25} = 1$

   c. $x^2 - y^2 = 1$

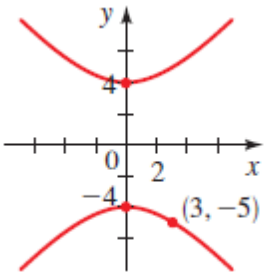
   d. $25y^2 - 9x^2 = 225$

5. Find an equation for the hyperbola whose graph is shown:

   a.



   b.

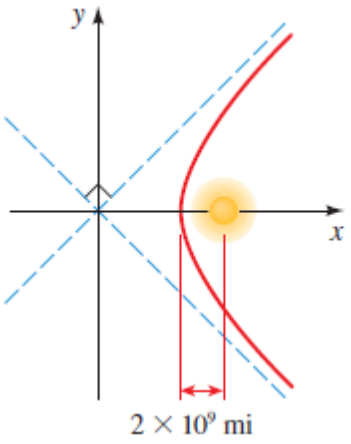


6. Find an equation for the hyperbola that satisfies the given conditions.
   a. Foci: $(\pm 5\ ,\ 0)$, Vertices: $(\pm 3\ ,\ 0)$
   b. Foci: $(0\ ,\ \pm 2)$, Vertices: $(0\ ,\ \pm 1)$
   c. Foci: $(\pm 1\ ,\ 0)$, Asymptotes: $y = \pm 5x$.
   d. Asymptotes: $y = \pm x$; hyperbola passes through $(5\ ,\ 3)$
   e. Foci: $(\pm 5\ ,\ 0)$; length of transverse axis: 1

II. Problem Solving

1. **Navigation:** In the figure below, the LORAN stations at A and B are 500 mi apart, and the ship at P receives station A's signal 2640 microseconds (μs) before it receives the signal from station B.
   a. Assuming that radio signals travel at 980 ft/μs, find $d(P, A)$ - $d(P, B)$
   b. Find an equation for the branch of the hyperbola indicated in red in the figure. (Use miles as the unit of distance.)
   c. If A is due north of B and if P is due east of A, how far is P from A?

2. **Comet Trajectories** Some comets, such as Halley's comet, are a permanent part of the solar system, traveling in elliptical orbits around the sun. Other comets pass through the solar system only once, following a hyperbolic path with the sun at a focus. The figure below shows the path of such a comet. Find an equation for the path, assuming that the closest the comet comes to the sun is 2×10^9 mi and that the path the comet was taking before it neared the solar system is at a right angle to the path it continues on after leaving the solar system.



III. Reasoning

1. Write an equation for a hyperbola where the distance between the foci is twice the length of the transverse axis.

2. Consider $rx^2 = -sy^2 - t$. Describe the type of conic section that is formed for each of the following. Explain your reasoning.
   a. $rs = 0$
   b. $rs > 0$
   c. $r = s$
   d. $rs < 0$

IV. Modeling

1. **Multiple Representations**. In this problem, you will explore a special type of hyperbola called a conjugate hyperbola. This occurs when the conjugate axis of one hyperbola is the transverse axis of another.
   a. **Graphical:** Sketch the graphs of $\frac{x^2}{36} - \frac{y^2}{64} = 1$ and $\frac{y^2}{64} - \frac{x^2}{36} = 1$ on the same coordinate plane.
   b. **Analytical**: Compare the foci, vertices, and asymptotes of the graphs.
   c. **Analytical:** Write an equation for the conjugate hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$.
   d. **Graphical:** Sketch the graphs of the new conjugate hyperbolas.
   e. **Verbal:** Make a conjecture about the similarities of conjugate hyperbolas.