2.3: The Human Cannonball

Parabola Equations and Graphs

As a human cannonball Rosa is shot from a special cannon. She is launched into the air by a spring. Rosa lands in a horizontal net 150 ft. from the cannon.

The equation $h = -0.0112\ x^2 + 1.84x + 5$ represents Rosa's height above the ground in feet, when she has traveled a horizontal distance of x feet. Rosa's launch will be filmed in 3-D with two cameras. Camera 1 is stationary and positioned under the vertex of her flight path. Camera 2 moves along a horizontal cable above Rosa's flight path. The cameras must always be the same distance from Rosa while filming. Camera 2 is mounted on a 58.250 ft. high stand, at a horizontal distance of 82.143 feet from the launch point.

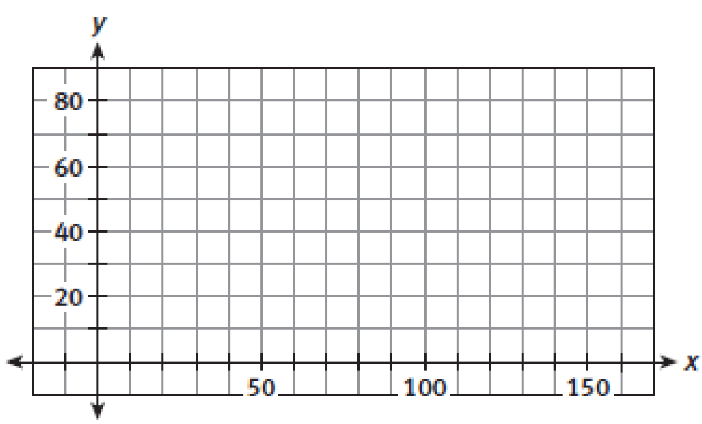
1. Describe the graph of $h = -0.0112\ x^2 + 1.84x + 5$.

2. Suppose that point P corresponds to the point where Rosa is directly above camera 2. Find the coordinates of point P and explain the meaning of each coordinate of the point.

3. When Rosa is at point P, how high must the cable be secured so the cameras will be the same distance apart?

4. Write an equation for the horizontal line representing the height of the cable for Camera 1.

5. Sketch a graph of the human cannonball situation. Graph the line representing Camera 1's movement, and label the point at which Camera 2 is located.

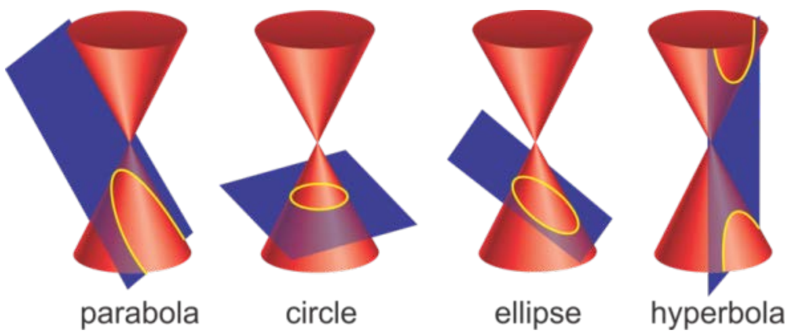


6. When Rosa has traveled a horizontal distance of 20 feet, how far is each of the cameras from her?

7. Show how this is true for two other horizontal distances that Rosa must travel.

Part 2: Investigating Conic Sections

In previous units you worked with quadratic functions. The graph of a quadratic function is a **parabola**. A parabola is one of the four conic sections studied by Apollonius, a 3rd century BCE Greek mathematician.



Conic sections are the curves we get when we make a straight cut in a cone, as shown in the figure above. For example, if a cone is cut horizontally, the cross section is a circle. So a circle is a conic section. Other ways of cutting a cone produce parabolas, ellipses, and hyperbolas.
In the next part of this lesson, you will explore how the different conic sections are formed. Don't worry we will return to Rosa, the Human Cannonball!

Open the interactive conic section located at http://ggbtu.be/mqZ8aGDzR and move the sliders to create the four different conic sections. The middle window is the cross section, and the sliders are located all the way in the right. If needed, you can rest the entire file by clicking on the blue double arrows in the upper right hand corner.

8. Describe, in detail, what conditions result in each of the four conic sections:
    a. Circle
    b. Ellipse
    c. Parabola
    d. Hyperbola

Open the interactive parabola located at http://ggbtu.be/muhqxQUW3 and move the slider to draw different parabolas.

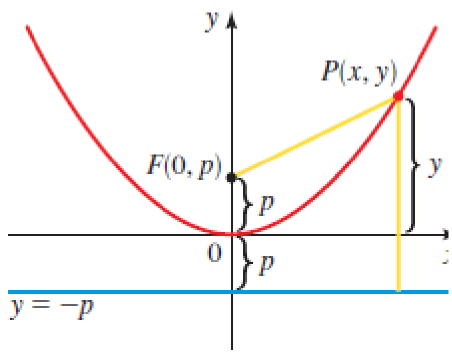
9. In your own words, what is the focus of a parabola?

10. In your own words, what is the directrix of a parabola?

11. What do you notice about the measurements of the two lines connecting to the pencil?

12. Define a parabola in terms of its focus and directrix: "A parabola is the set of points ____."

Part 3: The Equation of a Parabola



If *P*(x,y) is any point on the parabola, then the distance from P to the focus F (using the Distance Formula) is $\sqrt{x^2 + (y-p)^2}$

The distance from P to the directrix is |y-(-p)| or |y+p|.

By the definition of a parabola these two distances must be equal, so:

$$\sqrt{x^2 + (y-p)^2} = |y + p|$$

$$x^2 + (y-p)^2 = |y+p|^2 = (y+p)^2 \qquad \text{Square both sides}$$

$$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2 \qquad \text{Expand}$$

$$x^2 - 2py = 2py \qquad \text{Simplify}$$

$$x^2 = 4py$$

Conclusions: If $P(x,y)$ is any point on the parabola, then

- The graph of the equation is $x^2=4py$

and it is a parabola with the following properties:

- Vertex: $V(0,0)$
- Focus $F(0,p)$
- Directrix: $y= -p$

13. Find an equation for the parabola with focus $F(-8, -1)$ and directrix $y = -4$, and sketch its graph.

Part4: Rosa's Return

14. For the human cannon ball context what represents the parabola, the focus and directrix?

15. Sketch a graph of $y = x^2$.

16. Form the inverse relation by exchanging x and y and use your knowledge of the properties of inverses to sketch a graph of this relation on the graph in Question 15.

17. For each parabola, write the inverse relation and then sketch the original parabola and its inverse.
    a. $y = x^2 + 2$
    b. $y = (x + 1)^2$
    c. $y = -2(x - 3)^2$
    d. $y = \frac{1}{2}(x-1)^2 + 3$

    The inverse relations you graphed in Item 17 are parabolas with a horizontal axis of symmetry.

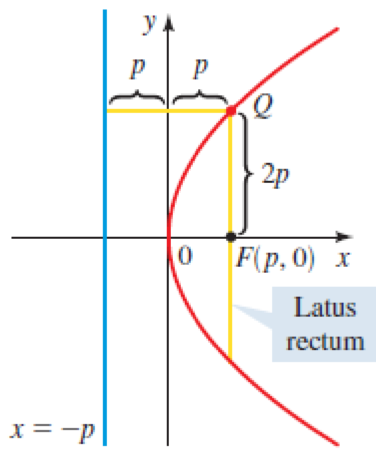
18. Sketch and label the axis of symmetry for each parabola you graphed in Question 17.

19. Label the coordinates of the vertex of each parabola you graphed in Question 17.

20. How can you determine whether or not a parabola has a vertical or horizontal axis of symmetry?

21. Write the equation and sketch the graph of the inverse of Rosa's Cannonball function: $h = -0.0112\,x^2 + 1.84x + 5$

Part 5: Focal Diameter



We can use the coordinates of the focus to estimate the "width" of a parabola when sketching its graph. The line segment that runs through the focus perpendicular to the axis, with endpoints on the parabola, is called the *latus rectum*, and its length is the focal diameter of the parabola. From the figure above we can see that the distance from an endpoint Q of the latus rectum to the directrix is $|2p|$. Thus the distance from Q to the focus must be $|2p|$ as well (by the definition of a parabola), so the focal diameter is $|4p|$. In the next example we use the focal diameter to determine the "width" of a parabola when graphing it.

22. Find the focus, directrix, and focal diameter of the parabola $= \frac{1}{2}x^2$, and sketch its graph.