Big Idea 4: Series (BC)

The AP Calculus BC curriculum includes the study of series of numbers, power series, and various methods to determine convergence or divergence of a series. Students should be familiar with Maclaurin series for common functions and general Taylor series representations. Other topics include the radius and interval of convergence and operations on power series. The technique of using power series to approximate an arbitrary function near a specific value allows for an important connection to the tangent-line problem and is a natural extension that helps achieve a better approximation. The concept of approximation is a common theme throughout AP Calculus, and power series provide a unifying, comprehensive conclusion.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 4.1: The sum of an infinite number of real numbers may converge.	**LO 4.1A:** Determine whether a series converges or diverges.	**EK 4.1A1:** The *n*th partial sum is defined as the sum of the first *n* terms of a sequence.
		EK 4.1A2: An infinite series of numbers converges to a real number S (or has sum S), if and only if the limit of its sequence of partial sums exists and equals S.
		EK 4.1A3: Common series of numbers include geometric series, the harmonic series, and *p*-series.
		EK 4.1A4: A series may be absolutely convergent, conditionally convergent, or divergent.
		EK 4.1A5: If a series converges absolutely, then it converges.
		EK 4.1A6: In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the *n*th term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.
		EXCLUSION STATEMENT (EK 4.1A6): *Other methods for determining convergence or divergence of a series of numbers are not assessed on the AP Calculus AB or BC Exam. However, teachers may include these topics in the course if time permits.*

CURRICULUM FRAMEWORK

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Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 4.1: The sum of an infinite number of real numbers may converge. *(continued)*	**LO 4.1B:** Determine or estimate the sum of a series.	**EK 4.1B1:** If a is a real number and r is a real number such that $\lvert r\rvert < 1$, then the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.
		EK 4.1B2: If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series.
		EK 4.1B3: If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value.
EU 4.2: A function can be represented by an associated power series over the interval of convergence for the power series.	**LO 4.2A:** Construct and use Taylor polynomials.	**EK 4.2A1:** The coefficient of the *n*th-degree term in a Taylor polynomial centered at $x = a$ for the function f is $\frac{f^{(n)}(a)}{n!}$.
		EK 4.2A2: Taylor polynomials for a function f centered at $x = a$ can be used to approximate function values of f near $x = a$.
		EK 4.2A3: In many cases, as the degree of a Taylor polynomial increases, the *n*th-degree polynomial will converge to the original function over some interval.
		EK 4.2A4: The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function.
		EK 4.2A5: In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the function.
	LO 4.2B: Write a power series representing a given function.	**EK 4.2B1:** A power series is a series of the form $\sum_{n=0}^{\infty} a_n (x-r)^n$ where n is a non-negative integer, $\{a_n\}$ is a sequence of real numbers, and r is a real number.
		EK 4.2B2: The Maclaurin series for $\sin(x)$, $\cos(x)$, and e^x provide the foundation for constructing the Maclaurin series for other functions.
		EK 4.2B3: The Maclaurin series for $\frac{1}{1-x}$ is a geometric series.
		EK 4.2B4: A Taylor polynomial for $f(x)$ is a partial sum of the Taylor series for $f(x)$.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 4.2: A function can be represented by an associated power series over the interval of convergence for the power series. *(continued)*	**LO 4.2B:** Write a power series representing a given function. *(continued)*	**EK 4.2B5:** A power series for a given function can be derived by various methods (e.g., algebraic processes, substitutions, using properties of geometric series, and operations on known series such as term-by-term integration or term-by-term differentiation).
	LO 4.2C: Determine the radius and interval of convergence of a power series.	**EK 4.2C1:** If a power series converges, it either converges at a single point or has an interval of convergence.
		EK 4.2C2: The ratio test can be used to determine the radius of convergence of a power series.
		EK 4.2C3: If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval.
		EK 4.2C4: The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series.