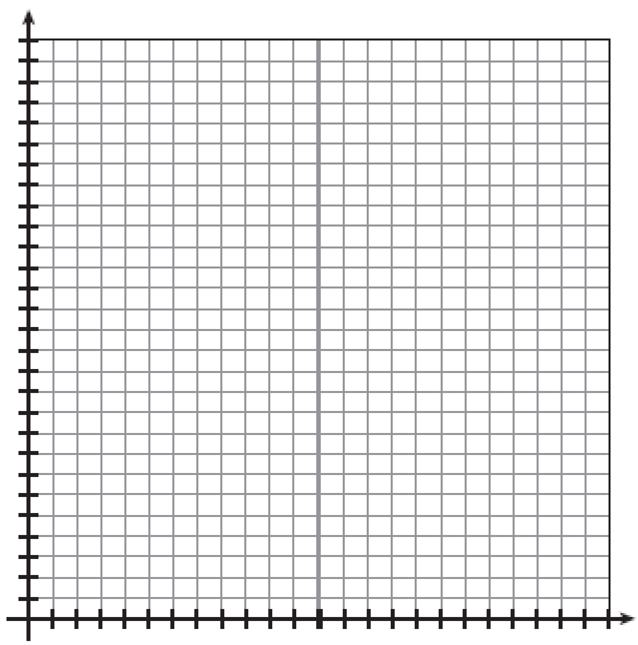
1.2: Close Enough

Introduction to Limits

Magdelena is out in the middle of the lake when her boat, the *Katie Ann*, breaks down. Two friends, Alex and Briana, who live on opposite sides of the lake, receive her calls for help and agree to come to her rescue.

1. Alex's movement can be described by the parametric equations $x = t$ and $y = \frac{1}{2}t + 4$. Briana's movement can be described by the parametric equations $x = -\frac{1}{2}t + 3$ and $y = -t + 7$. In both cases, t represents hours traveled.
   a. Plot the starting location for each of Magdelena's friends on the grid below.



   b. Both friends travel half the distance to Magdelena in 1 hour. Determine the coordinates for the location of each friend after 1 hour. Plot these coordinates on the grid and trace each friend's path.

   c. From their locations after 1 hour, Alex and Briana use an additional half hour to travel half the remaining distance to Magdelena. Use $t = 1.5$ to determine the coordinates for the location of each friend after 1.5 hours. Plot these coordinates on the grid and trace each friend's path.

d. Again, both friends travel half the remaining distance to Magdelena. This time the friends travel an additional quarter of an hour. Use $t = 1.75$ to determine the coordinates for the location of each friend. Plot these coordinates on the grid and trace each friend's path.
e. A fourth time, Alex and Briana travel half the remaining distance to Magdelena, using an additional one eighth of an hour. Use $t = 1.875$ to determine the coordinates for the location of each friend. Plot these coordinates on the grid and trace each friend's path.
f. Predict the coordinates for the location of Magdelena and her boat, the *Katie Ann*. Explain your reasoning.
g. Assuming Magdelena's friends continue to move half the distance from their locations to her boat, will they ever arrive at her exact location? Explain your reasoning.

II. Limits

The concept represented by Magdelena and her friends is the idea of a limit. If an object is approached from both sides, it is possible to eventually be close enough to the object to give its exact location.

In mathematics, it is common to discuss the *limit of a function*. In order to understand the concept of a limit, you will begin by using several methods to examine functions near a certain point.

2. Let $f(x) = \frac{x^2-1}{x-1}$. Use the following methods to explore the values of y as the value of x approaches 1.
   a. Complete the table below.

x approaches 1 from the left →   ← *x approaches 1 from the right*

x	**0.9**	**0.99**	**0.999**	**1**	**1.001**	**1.01**	**1.1**
f(x)							

   b. Use a graphing calculator to graph $f(x)$. Use the trace feature of your calculator to examine the values of y as the value of x approaches 1 from both directions. What happens to the values of y as the value of x approaches 1?

   c. What seems to be true about the values of y for $f(x)$ as the values of x approach 1? Do both representations of the function (table and graph) lead you to the same conclusion?

3. Let $g(x) = x^2 + 2$.
   a. Complete the table below.

x approaches 1 from the left → ← *x approaches 1 from the right*

x	**0.9**	**0.99**	**0.999**	**1**	**1.001**	**1.01**	**1.1**
f(x)							

   a. Use a graphing calculator to graph $g\ (x)$. Use the trace feature of your calculator to examine the values of y as the value of x approaches 1 from both directions. What happens to the values of y as the value of x approaches 1?

   b. What seems to be true about the values of y for $g\ (x)$ as the values of x approach 1? Do both representations of the function (table and graph) lead you to the same conclusion?

4. Let $h(x) = \frac{1}{x-1}$.
   a. Complete the table below.

x approaches 1 from the left → ← *x approaches 1 from the right*

x	**0.9**	**0.99**	**0.999**	**1**	**1.001**	**1.01**	**1.1**
f(x)							

   b. Use a graphing calculator to graph $h(x)$. Use the trace feature of your calculator to examine the values of y as the value of x approaches 1 from both directions. What happens to the values of y as the value of x approaches 1?

   c. What seems to be true about the values of y for $h(x)$ as the values of x approach 1? Do both representations of the function (table and graph) lead you to the same conclusion?

III. Terminology and Definition

The **limit** of a function $f(x)$ is the one number L that $f(x)$ becomes arbitrarily close to as x approaches, but does not equal, a number c. In other words, when a function has a limit; as x approaches a value c, the y value of the function $f(x)$ approaches the limit value, L.

The notation $\lim_{x \to c} f(x)$ is read as "the limit of $f(x)$ as x approaches c."

5. Use the functions in items 2, 3, and 4 to evaluate the following, if they exist.

   a. $\lim_{x \to 1}(x^2 + 2) =$

   b. $\lim_{x \to 1} \frac{x^2-1}{x-1} =$

   c. $\lim_{x \to 1} \frac{1}{x-1} =$

IV. Practice – Khan Academy

Complete the six online practice exercises in the first unit (Limits and Continuity) of Khan Academy's AP Calculus AB course:

- https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-continuity/ab-limits-graphically/e/two-sided-limits-from-graphs
- https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-continuity/ab-limits-graphically/e/connecting-limits-and-graphical-behavior
- https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-continuity/ab-limits-numerically/e/creating-tables-to-approximate-limits
- https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-continuity/ab-limits-numerically/e/finding-limits-numerically