Individual Work 5.2: Those Moose Were Made For Walkin'

In this set of exercises, you revisit the properties of additive processes in general, and contexts such as the moose migration that form additive processes. In addition, stretch and shift transformations are applied to scales and to the equations that work with them.

1. Write a closed-form linear equation that is consistent with each of the following statements.

   a) $x(3) = 5$, $x(9) = 7$

   b) $T(10) = 50$, $T(20) = 68$

2. **Figures 5.5–5.7** show three data tables. Explain whether each represents an additive process.

   a)

Century	Population
0	120
1	124.7
2	129.4
3	134.1
4	138.8
5	143.5
6	148.2

Figure 5.5. Population data.

   b)

Day	Population
0	1
1	5
2	13
3	25
4	41
5	61
6	85

Figure 5.6. Population data.

c)

Year	Population
0	180
1	201
2	226
3	253
4	283
5	317
6	355

Figure 5.7.
Population data.

3. Here is an additive process that has been described by a recursive equation:

$$x_n = x_{n-1} + 3;\ x_0 = 8$$

a) Make a data table like the one shown in **Figure 5.8,** and record 5 rows of values, using this recursive equation.

x_{n-1}	x_n
:	:

Figure 5.8.

b) Recall that a recursive graph is one that is made by pairing values of the form (previous value, current value). Make a recursive graph of the data in your table from part (a) and describe the features of the graph.

c) What closed-form equation describes that same additive process?

d) Make a graph of x versus n, using the closed-form equation from part (c). Describe the features of that graph.

4. Reflect back on the work that was done in Question 3 to describe an additive process recursively and in closed-form.

a) How can you tell the slope and y-intercept of the recursive graph from the recursive equation?

b) How can you tell the slope and y-intercept of the graph of the closed-form equation from the recursive equation?

c) How could you tell the recursive equation, from just looking at the recursive graph?

d) How could you tell the recursive equation, from looking at the graph of the closed-form equation?

5. The two graphs shown in **Figures 5.9–5.10** describe additive processes. One of them is a recursive graph, and the other one is the graph of a closed-form equation.

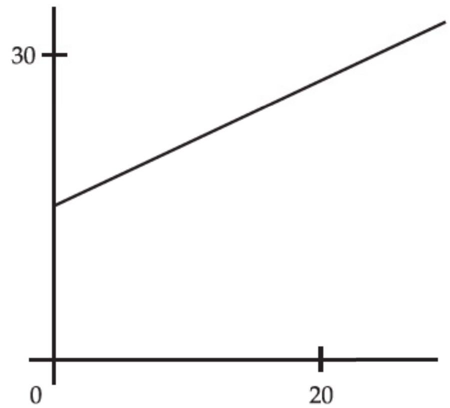


Figure 5.9.

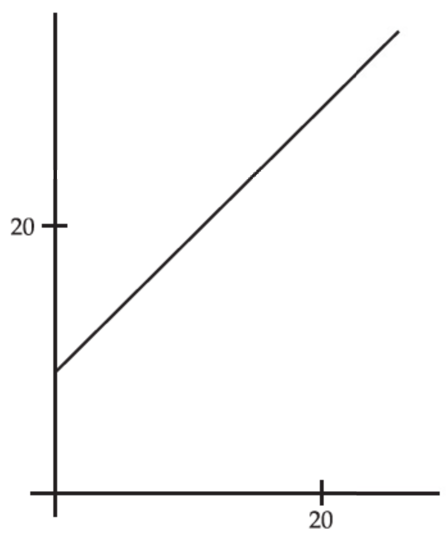


Figure 5.10.

Unfortunately, the axes have not been labeled. From the graphs alone, determine which one is the recursive graph, and which one is the graph of the closed-form equation. Explain how you arrived at your answers.

6. JaMelle wants to make a model to describe the moose population growth, starting with 1993 as Year 0. (That timeline decision becomes an assumption for her model!) She examined the given information and made the following assumptions.

   - Population changes only by migration of new male moose into the park.

- The same number of moose arrives each year.
- The migration rate is the slowest that is also consistent with the 1988 and 1993 observations.
- There are no deaths.
- No additional moose are brought into Adirondack State Park.

a) What population value did JaMelle assume for 1988 and 1993? Write your answer as both ordered pairs and in function notation.

b) What migration rate did she use? Be sure to include proper units.

c) Write a recursive equation to describe the current population in terms of the previous year's population.

d) Write a closed-form equation to describe the current population in terms of the number of years that has elapsed.

7. JaMelle wants to make graphs that describe the population growth under her assumptions.

a) Make a three-column data table (year number, previous and current population) for her model from 1993 to 2000.

b) Make a recursive graph for her model.

c) Make a closed-form graph for her model.

d) If you project her model into the future, what would the moose population be in the year 2050?

8. Mark decided to model the moose population using the same assumptions as JaMelle, but with the following change:

- The migration rate is the fastest that is also consistent with the 1988 and 1993 observations.

a) Build this model, recursively and in closed form, using graphs and tables wherever appropriate.

b) Use your model to predict the moose population in 2050, and then use home-screen iteration to verify that calculation.

9. Each of the following is a set of control numbers for the work done in Activity 5.2. Construct closed-form equations to model each scenario. Be sure to describe the conditions for using the equation.

a) Model begins with 1988 as Year 0, $P(0) = 15$ moose, migration rate = 1 moose/year, no additional moose added.

b) Model begins with 1993 as Year 0, $P(0) = 27$ moose, migration rate = 3 moose/year, 100 additional moose added in 1993.

c) Model begins with 1993 as Year 0, $P(0) = 25$ moose, migration rate = 2 moose/year, 40 additional moose added in 1993 and 60 more moose added in 1995. (Note: this will require two equations to completely describe.)

10. Blue whales weigh about 3 tons at birth, and grow at a rate of about 200 lbs. per day during the first months of life. **Figure 5.11** shows a portion of the growth records kept on a single whale by two brothers, Cale and Dale.

Age (days)	Weight (lb)
0	6000
1	6200
2	6400
3	6600
4	6800
5	7000

Age (weeks)	Weight (lb)
0	6000
1	7400
2	8800
3	10,200
4	11,600
5	13,000

Figure 5.11. Records of whale weights using different time scales.

a) Copy the first set of data and add another column for the change in weight. Is it an additive model? Explain.

Cale made a closed-form graph for that same data. He added little vertical segments to the points of the graph, creating what looks like signposts as shown in **Figure 5.12.**

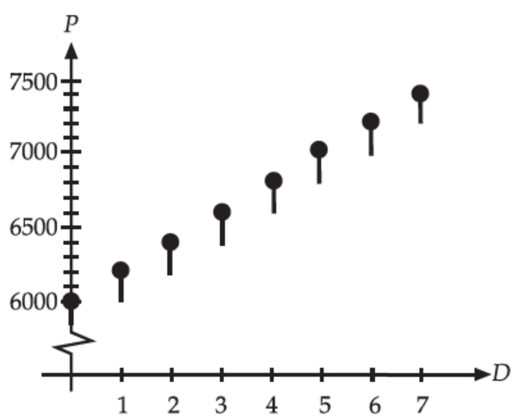


Figure 5.12.

b) What do the lengths of the signposts tell you about this growth? Explain.

c) What is the slope of the line that goes through all of Cale's data points? Be sure to include any units.

d) According to the pattern in the data that Dale has collected, in how many days will the whale reach a weight of 10,200 lbs.? Explain.

11. Dale thought the two data tables were consistent, until he calculated the rate of change for the second table.

a) What is the rate of change for the second data table?

b) Use unit-conversion methods to relate the slopes of the two data sets.

In Activity 5.2, one of the decisions that you made determined the year in which the model began. When testing the model, it was the only control number not investigated. However, revisiting that decision gives you the opportunity again to explore an interesting area related to equations.

12. Two different student groups had the following migration models: $P = 2n + 20$ and $P = 2t + 30$. Both models predict the same moose population for the park in 2013. The difference between the two groups is that one model was built to start in 1988, while the other started in 1993.

a) Verify that the models give the same prediction for the 2013 population. Show your work.

b) What is the relationship between the variables t and n?

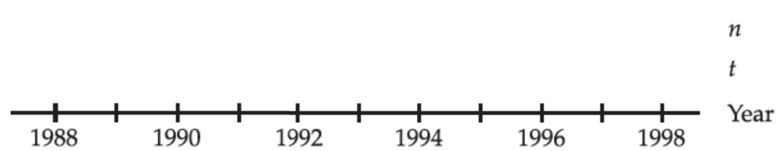
c) Start with the equation $P = 2t + 30$ and use your relationship from part (b) to replace the variable t with an expression equal to it. What new equation is produced?

d) Show that this is algebraically equivalent to the first model.

13. There are many ways of thinking about what you did in Question 12, and why it works. Here are several representations.

a) **Figure 5.13** shows a timeline for the calendar years 1988 to 1998.

Figure 5.13.



Make a copy of the figure on your paper, and fill in scales that correspond to the two variables—n (number of years since 1988) and t (number of years since 1993). How would you "shift" the n-scale to line up the same numbers with the t-scale?

b) **Figure 5.14** is a table that shows the calculations involved in the equation $P = 2t + 30$ for the 5-year period that begins with 1993, showing the order of operations.

n	t	$2t$	$2t + 30$
	0	0	30
	1	2	32
	2	4	34
	3	6	36
	4	8	38
	5	10	40

Figure 5.14.

Fill in the column of values for the variable n that corresponds to each value of t. How would you "shift" the column of t-values so that the numbers line up with the column of t-values?

c) **Figure 5.15** shows the closed-form graphs for the two migration models.

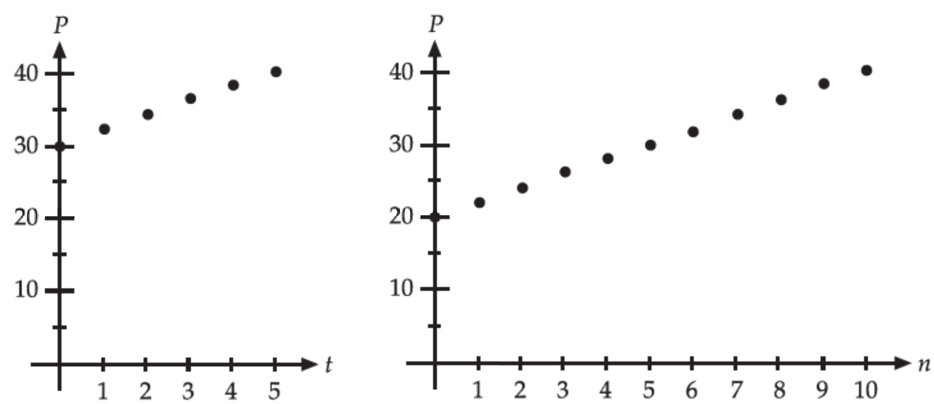


Figure 5.15.

Trace a copy of the graph of $P = 2t + 30$, complete with axes and scales. Place your drawing over the graph of $P = 2n + 20$ so that the axes line up with each other. How far over (to the left or right) must you shift your graph until the data points line up?

The work that was done in Questions 12–13 began with two models that used different explanatory variables. The question, *how can I compare the two models*? was answered by expressing one model in terms of the explanatory variable used in the other. You did this by substitution, replacing the variable with an expression that was equal to it. The expression said, in mathematical terms, to take one variable and add (or subtract) something, in order to get the other variable. Doing this shifted values of the explanatory variable, and also moved the graph to the right (or left). The shifting of a graph in the same direction as the x-axis is called a **horizontal translation**. One way to understand translations is by working with parametric equations.

14. You would like to know what equation describes the graph that is made by translating the graph of the equation $y = 2x + 3$ four units to the right. The table in **Figure 5.16** shows some of the values before, and after, the translation.

Figure 5.16.

Before translation			After translation		
t	x	y	t	x	y
0	0	3	0	4	3
1	1	5	1	5	5
2	2	7	2	6	7

a) How can you tell from the table values that a horizontal translation has taken place?

b) The recursive programming command, $x + 4 \rightarrow x$, can be used to animate a pixel horizontally across the calculator screen. Explain how this can be interpreted as a horizontal translation of a single point.

c) Write parametric equations to describe the original equation $y = 2x + 3$ (before the translation). Check your answer by substituting out the parameter variable. Do you get back the original equation? Explain.

d) Now, write parametric equations to describe the new equation (after the translation). Use substitution to eliminate the parameter variable. What new equation do you get? How does it compare to the original equation?

In Chapter 1, *Secret Codes*, you studied shifts as a process in which a number is added (or subtracted) after a "stretch" transformation. This had the effect of translating the graph upward (or downward). What you are now exploring is another type of shift, caused by adding or subtracting a number to the variable before any stretching action. This has the effect of translating the graph horizontally—either to the right or the left.

15. Imran really enjoyed using parametric equations in Chapter 4, *Animation*. He wants to translate a line horizontally, but while the equation is written in parametric form.

a) Write parametric equations for the equation $y = 3x + 5$, that was written in slope-intercept form.

b) Write new parametric equations that would have move the graph of his line 4 units to the left. Graph and trace the equations to verify that corresponding points really have been translated exactly 4 units to the left.

c) Use substitution to convert your answer into a closed-form linear equation. Graph your new equation to verify that it has the same graph as the parametric equations from which it came.

16. For each of the pairs of graphs in **Figures 5.17–5.19**, explain how their equations must be related.

a)

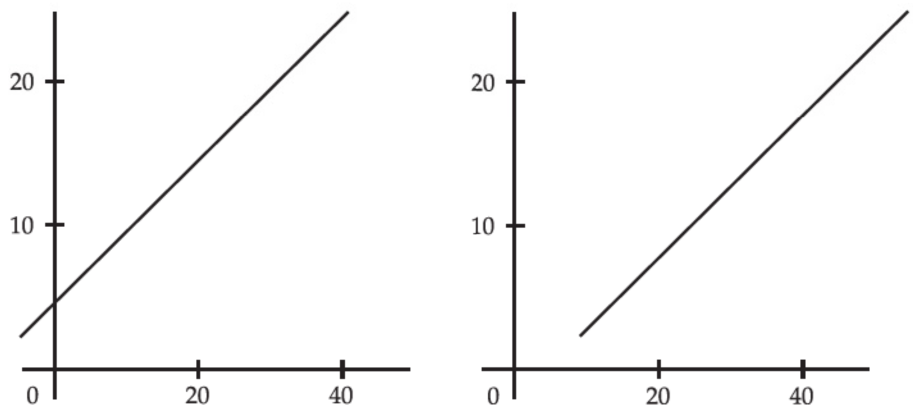


Figure 5.17.
Two lines.

b)

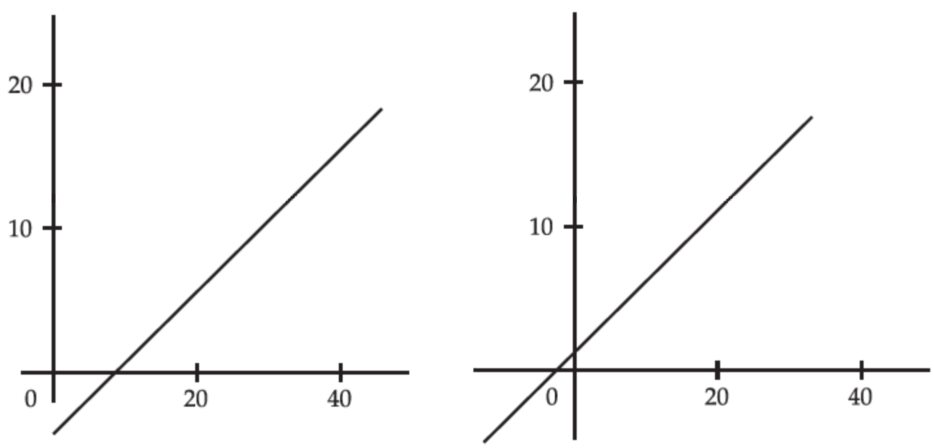


Figure 5.18.
Two lines.

c)

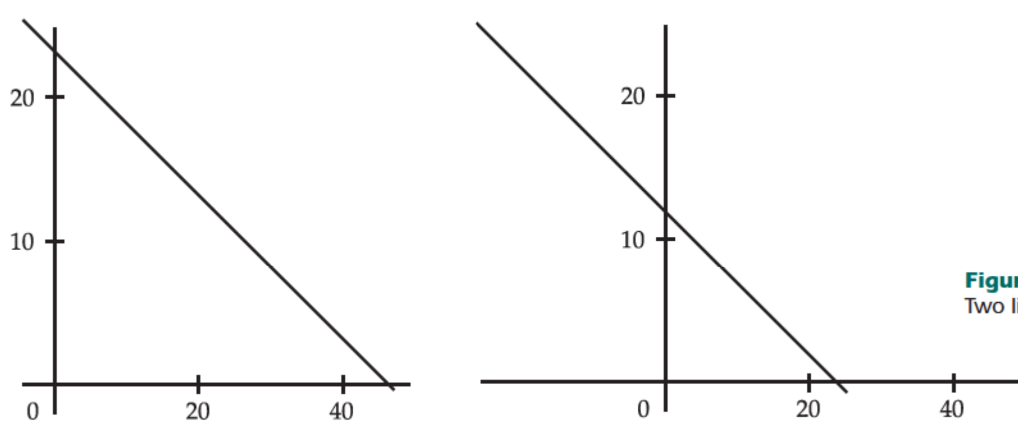


Figure 5.19.
Two lines.

The shifts that were studied in Chapter 1, *Secret Codes*, can be considered **vertical translations** of the graph of a direct variation. The same treatment that was done to the explanatory variable in Question 14 can be done to the response variable.

17. You want to know what equation describes the graph that is made by translating the graph of the equation $y = 2x$ up three units.

    a) Write parametric equations to describe the original closed-form equation $y = 2x$.

    b) Now, write parametric equations to describe the new graph that was made by translating the original one upward three units.

    c) What closed-form equation is formed, when you eliminate the parameter variable by substitution?

In Chapter 3, *Predictions*, you wrote equations for lines in point-slope form. One way of viewing the information contained in a point-slope equation form is that "a line with a particular slope goes through a point whose coordinates are given." It is also possible to interpret the same information using the idea of a translation.

18. Consider the equation: $y - 3 = 2(x + 4)$, which is written in point-slope form. Think of it as an equation of the form $y = mx$ that has been translated both horizontally and vertically.

    a) Before the translations, with what equation would you need to start? Explain.

    b) How much of a horizontal translation, and in what direction, would the graph of the direct variation need to undergo? How can you tell from looking at the equation?

    c) How much of a vertical translation, and in what direction, would it need? How can you tell from looking at the equation?

    d) Sketch the original direct variation graph, then use your answers to parts (b) and (c) to translate that graph into its new position. Sketch the graph that is produced by the two translations (which is the graph of the original equation).

19. Write closed-form equations for each of the indicated translations of the given equations.

    a) $y = 3x - 4$. Translate to the right 3 units.

    b) $y = 1.5x + 7$. Translate to the left 2 units.

    c) $3y + 2x = 10$. Translate to the right 1 unit.

    d) $y = 2x$. Translate to the right 3 units and down 4 units.

20. You can also use the idea of translating a line from the origin to a point to find the equation of the line that contains two given points.

a) Find the slope of the line going through the points (3, 7) and (9, 21).

b) Imagine a line going through the origin that has the same slope as what you found in part (a). What equation would describe that line?

c) Now, translate that line to the point (3, 7). What new equation will be formed?

d) Now, repeat the steps from parts (a)–(c), and find the equation of the line that goes through the points (4, 28) and (8, 18).

The idea of applying transformations to axis scales may be new, but after all, scales are just numbers. You have been applying shift transformations throughout this course. You can also stretch axis scales; in fact, you did it in this set of exercises!

21. Recall Cale and Dale, the two whale researchers from Questions 10–11. One kept growth information on a daily basis, while the other kept it on a weekly basis.

a) Write closed-form equations for the growth models describing each data set.

b) What is the relationship between the two independent variables? Express this as an equation.

c) Start with the model that uses the number of days elapsed to predict the baby whale's weight. Use the relationship you just developed in part (b), and replace the independent variable with an expression that is equal to it. What new equation is produced?

d) Show that this new equation is algebraically equivalent to the model that predicts the baby whale's weight from the number of weeks elapsed.

This kind of transformation is known as a **scale change transformation**.

22. Suppose the two boys' sister, Gail, was also recording data on the weight of a baby whale. But she only recorded data every month (1 month = 4 weeks).

a) If you compared the graphs of the three closed-form equations (P vs D, P vs W, and P vs M), which would be steepest? Explain.

b) Use a scale change transformation to rewrite the model that uses the number of weeks elapsed to predict the weight, so that it uses the number of months elapsed instead. What new equation is produced?

c) Verify that the new equation correctly predicts the weight of a baby whale by finding P(4 weeks) and P(28 days), and comparing them to P(1 month).

23. Write closed-form equations for each of the indicated scale change transformations of the original equation given. (Check by graphing both the given equation and the transformed equation in the same window.)

    a) $y = 3x - 4$. Make it 1.5 times wider.

    b) $y = 1.5x + 7$. Make it 4 times narrower.

As part of the modeling process, Activity 5.2 asked you to determine the control number to which your migration model was most sensitive. Now apply sensitivity analysis to a model that was developed earlier in the course.

24. Recall that you built a model in Chapter 3, *Predictions*, for predicting the number of manatee deaths. The equation was $y = 0.125x - 41.4$, where x was the number of powerboats registered (in thousands) and y was the number of manatee deaths.

    a) Use the equation to predict the number of manatee deaths for $x = 425$, $x = 575$, and $x = 725$. Round your answers to the nearest integer.

    b) Test the sensitivity of these predictions to changes in the y-intercept value. Replace the –41.4 in the equation with –33. (This is about a 20% increase). What will the new equation predict as the number of manatee deaths for the same x-values?

    c) Test the sensitivity of the original predictions to changes in the slope. Use –41.4 as the y-intercept but replace the slope with 0.15 (representing a 20% increase). What will the new equation predict as the number of manatee deaths for those same x-values?

    d) Summarize your work from parts (b) and (c). Which seems to have the larger effect on the predictions—changes in the slope or in the y-intercept?