Activity 5.2: My Great Model

In this activity, you develop a model to describe the moose population growth, assuming that it is caused by migration. In addition, you test that model to determine if environmentalists used it in predicting 1300 moose in the park in the year 2013.

PART I: BUILDING THE MODEL

Building models is never as easy as grabbing a calculator and punching some buttons. Careful thought must go into the beginning stages of modeling. Questions such as, *what makes sense to do*? *what should you try first*? *is this really what is going on*? are ones that help you take your first steps. However, at some point you have to roll up your sleeves and proceed to Step 3 of the modeling process.

> **Step 3:** Build the model: Interpret, in mathematical terms, the features and relationships you have chosen. (Define variables, write equations, draw shapes, gather data, measure objects, calculate probabilities, etc.) Identify properties that are mathematical consequences of the model.

You're going to build your first model and figure out how many moose will be in the park in the year 2013. But first, you need to make some decisions about the control numbers for the problem. Recall that there are two things known about the moose population in Adirondack State Park. There were between 15 and 20 moose in the park in 1988, and it increased to somewhere between 25 and 30 moose in 1993.

1. Recall that the response variable is another name for what you are trying to predict, and the explanatory variable is what you use to make that prediction. What should be the explanatory and response variables for the moose migration model?

2. Your model must begin at some point in time, and then you project population growth from that moment.

    a) Which year would you like to use as the beginning point for your model? (From now on, you can call that Year 0 for your model.)

    b) Eventually, you want to use your model to predict the moose population in the year 2013. How many years in the future is that from your beginning point?

Recall from the work done in Individual Work 5.1 that the function notation $C(2)$ meant the cost of buying two pencils. The rule describing *how* to determine that cost was called the function $C(n)$, where n was the number of pencils.

3. We will often use similar function notation in this activity, with the model that you create being called $P(t)$, where t is the number of years that have elapsed since your model began.

   a) What would the notation $P(0)$ and $P(1)$ mean for your model?

   b) According to the facts, you have a range of possible values for the starting population for your model. What specific value will you assume for $P(0)$? Why?

You have already made two major decisions in building your model. You introduced a timeline by deciding which year in which to begin the model. You also assumed a starting population that was consistent with the known facts at that time. The third decision you must make has to do with controlling how many moose migrate each year. As another way of simplifying the problem, assume that the same number of moose arrive each year. Then your model will be based on a constant migration rate. You still need to determine what value to use for that rate.

MODELING NOTE

When having to work with an unknown quantity, modelers have two basic options—make it a variable or assign it a specific value within reason. When you choose a particular value, as you did in the previous question, it becomes an additional assumption for the model.

The known facts about the moose populations are shown in a graphical way in **Figure 5.4**. The bars are used to indicate a range of possible values for the population estimate.

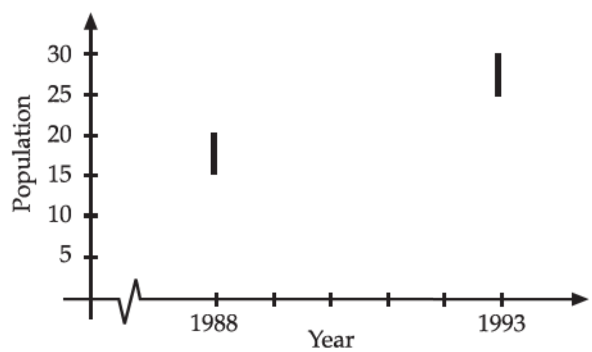

Figure 5.4.

TAKE NOTE

Remember that the key feature for this model is that we assume *all* changes in the moose population are due to moose migrating into the park.

4. Consider a scenario in which it is assumed that the population in 1988 was 17 moose, and in 1993 it was 27 moose.

   a) How many years elapsed during this time? What was the change in the size of the moose population over that time period?

   b) What is the rate of change for this scenario? What units describe this rate of change?

TAKE NOTE

Recall in Chapter 4, *Animation*, that a convenient way to compare two quantities changing at the same time is with a rate of change. You calculated (and described) it this way:

rate of change = $\frac{\text{change in \_\_\_\_\_\_}}{\text{change in \_\_\_\_\_\_}}$ or __________ per __________

c) Suppose the population change was entirely due to moose migrating into the park. In this scenario, how many moose would come into the park (on an average) each year?

d) Imagine a line going through the points (1988, 17) and (1993, 27) in Figure 5.4. What value would the slope of the line have? (Be sure to include units!)

In assuming that the same number of moose migrates into the park each year, the *type* of equation to use for your mathematical model has also been determined. A constant rate of change, like having the same number of moose arrives each year, is characteristic of linear equations. Besides the slope, the other number that determines a line is the y-intercept, the place where a time-series graph begins.

5. Refer back to the graph in Figure 5.4. Imagine a line having to touch each of the bars shown. (You may want to use a spaghetti noodle or the edge of a ruler.)

   a) What assumptions would you make for the 1988 and 1993 populations, so that the annual migration rate would be as small as possible?

   b) For the annual migration rate to agree with the facts, what would be its smallest possible value?

   c) What assumptions would you make for the 1988 and 1993 populations, so that the annual migration rate would be as big as possible?

   d) If the annual migration rate is to agree with the known facts, what would be its largest possible value?

MODELING NOTE

The control numbers for the migration model are the starting population, the year in which the model begins, the migration rate, and whether or not the 100 extra moose are added to the population. Each one is an assumption for your migration model. Together, the three decisions make up the conditions for a particular model.

6. According to the facts, you have a range of possible values for the migration rate to use with your model. What specific value will you assume for that control number?

7. Now, put it all together to make your migration model. In this case, assume that the extra 100 moose are not moved into the park.

   a) What does $P(1)$, $P(2)$, $P(3)$, and $P(4)$ equal? Record those values in a table.

   b) What equation describes your model?

   c) According to your model, what will the population be in the year 2013? Describe your answer using function notation.

   d) Recall that in recent years, the first moose sighting was after 1980. According to your model, when did the moose begin to migrate into the park?

8. Repeat the work that was done in Question 7, only this time assume that the extra 100 moose are moved into the park. To keep things simple, assume that they are all moved into the park in 1993.

   a) What do $P(1)$, $P(2)$, $P(3)$, and $P(4)$ equal? Record those values in a table.

   b) What equation describes your model?

   c) According to your model, what will be the population in the year 2013?

   d) Describe what the time-series graph would look like over the years from 1988 to 1998. If necessary, make a data table and plot the values on a set of axes.

Before continuing to the second part of this activity, exercise one more critical modeling skills. Communicate your findings with other groups, and compare their results with your own. Every set of control numbers that was used corresponds to a scenario that may have actually happened. You want to know what range of results is possible from your model.

PART II: TESTING THE MODEL

A model is not the answer to a problem; it is a means to finding that answer. In many ways, it is a concept that is used to represent a situation. Before you can claim that something behaves a certain way, you had better make sure that the model that describes it works! This is why there is a fourth (and last) step to the modeling process:

> **Step 4:** Evaluate and revise the model. Go back to the original situation and see if results of the mathematical work make sense. If so, use the model until new information becomes available or conditions change. If not, reconsider the assumptions you made in Step 2 and revise them to be more realistic.

One aspect of the process of testing a model is a form of error analysis. Are your predictions different from what you expected them to be, simply because of the values you chose for your parameters? Is it possible to choose different values that would give the answer you want?

To answer those questions, you work backwards, assuming that the answer is correct. You want to find out whether the conditions for getting that answer are within the restrictions of the model. Since you are testing your model by comparing it to the environmentalists' results, you should expect the answer to be 1300 moose by the year

> **TAKE NOTE**
> For our situation, *testing* the model will be comparing predictions from the migration model with ones from the environmentalists' model. You want to know if they were also using a migration model.

2013. So, use error analysis to decide if there is any way that your model can get that same answer.

9. Assume the largest starting population possible for the conditions of the problem. Go ahead and move the extra 100 moose into the park as well.

   a) If the herd grows to 1300 by 2013, what would the migration rate have to be?

   b) Is it possible to have the migration rate that big, considering the facts for this problem?

10. Assume the largest migration rate possible that is supported by the facts. Move the extra 100 moose into the park as well.

   a) If the herd grows to 1300 by 2013, what would the starting population have to be?

   b) Is it possible to have the starting population be that big, without contradicting the data?

11. Assume the largest migration rate and starting population that is possible. Spend $1.3 million and truck in an extra 100 moose. How many years will it take for the population of the herd to reach 1300 moose, according to your model?

12. Re-read Step 4 of the modeling process.

   a) Were the environmentalists using a migration model? Explain.

   b) As a mathematical modeler, what should you do next?

TAKE NOTE The descriptions in the next few questions use a standard mathematical symbol, Δ (the Greek letter delta), to mean "the change in." So, an expression like Δx is read "the change in x," or more simply "delta x." But it *means* that there are actually two values for the variable x, and you are interested in their difference.

Another way to test a model is to explore how sensitive it is to each control number. To do this, change only one value at a time, and only by a small amount. The control number to which the model is most sensitive will have the greatest effect on the prediction from the model.

13. Assume that you start with 25 moose in 1993, and that 2 additional moose migrate into the park each year.

   a) How many moose will be in the park in the year 2013?

   b) If you change the starting population by 1 moose ($\Delta SP = 1$), how much will the final population (in 2013) change? Call that answer (ΔP).

   c) Express your answer to part (b) as a rate of change—how much the final population increases for each additional moose in the starting population.

14. Assume the same initial conditions as in Question 13: that you start with 25 moose in 1993, and that 2 additional moose migrate into the park each year.

a) If you change the migration rate by 1 moose per year (ΔMR = 1 moose/yr), how much will $P(2013)$ change?

b) Express your answer to part (a) as a rate of change—how much the population increases for each additional moose included in the migration rate.

15. Again, assume the same initial conditions as in Question 13.

a) If you spend the money and add an additional 100 moose, how much will $P(2013)$ change?

b) Express your answer to part (a) as a rate of change—how much the population increases for each additional moose brought into the park.

16. To which of the control numbers—starting population, migration rate or whether to add the extra 100 moose—is the migration model the most sensitive?

Activity Summary

In this activity, you:

- were introduced to Step 3 (build the model) of the modeling process, where you:
  - — identified explanatory and response variables t and P, and made a decision about which year to use as Year 0 for the model.
  - — assumed values for the control numbers for the model—starting population, migration rate, and whether or not the additional 100 moose were moved into the park.
  - — used those numbers to build a specific migration model, described by a linear equation.
- were introduced to Step 4 (test the model) where you:
  - — tested that model to find out whether it was possible for the environmentalists to be using a migration model in their own prediction.
  - — analyzed the model to determine the control number to which it is most sensitive.

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DISCUSSION/REFLECTION

1. Suppose you wanted to assume a starting population of 20 moose in 1988 and a migration rate of 3 moose/year after that. Why is that inconsistent with the observed data?
2. What key assumption made the migration model be an additive process?
3. Based on the work in this activity, you should start considering some other factor, besides migration, to explain the future population growth. Does this mean that moose cannot have migrated into the park?

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