Homework 2 Investigating Inequalities

Part I: Manipulating Inequalities

In *Solve It!* you used the mystery bags game to think about ways to change equations but keep them true. For instance, if you had a true equation—that is, two expressions that were equal—you could add the same quantity to both sides of the equation, and the resulting expressions would still be equal.

For example, the statement $3 + 8 = 5 + 6$ is true, because $3 + 8$ and $5 + 6$ are both equal to 11. If you add 7 to both sides, the resulting statement is $3 + 8 + 7 = 5 + 6 + 7$ and this statement is also true.

1. The first aspect of Part I is to investigate whether similar principles hold true for inequalities. Start with the inequality $4 > 3$, which is true. For this inequality, perform each of these tasks and then examine whether the resulting statements are true.

   - Add the same number to both sides of the inequality.
   - Subtract the same number from both sides of the inequality.
   - Multiply both sides of the inequality by the same number.
   - Divide both sides of the inequality by the same number.

   For example, if you multiply both sides of the inequality $4 > 3$ by 2, the statement becomes $4 \cdot 2 > 3 \cdot 2$. Your task for each operation is to determine if the new statement is true no matter what "the same number" is.

   Try different possibilities for "the same number," using both positive and negative values.

2. After you finish working with the inequality $4 > 3$, start with a different true inequality and see whether you reach the same conclusions.

3. When you are done exploring, state your conclusions. Make them as general as possible.

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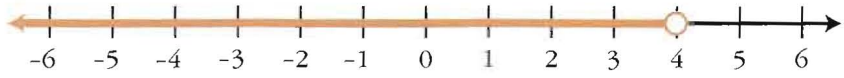
Part II: Graphing Inequalities

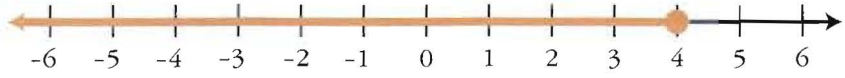
If an inequality has a single variable in it, we can picture all the numbers that make the inequality true by shading them on a number line. This is called the **graph of the inequality.** An inequality using $<$ or $>$ is called a **strict** inequality. An inequality using $\leq$ or $\geq$ is called **nonstrict.**

For example, the colored portion of this number line represents the graph of the strict inequality $x < 4$:



The open circle at the number 4 on the number line means that the number 4 is not included in the graph. (The number 4 is not included because substituting 4 for x gives a false statement.) *Note:* The exclusion of an endpoint is sometimes represented by a parenthesis instead of the open circle.

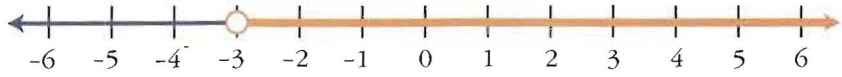
If we want to include a particular number as part of the graph, we mark that point with a filled-in circle (or by a bracket). For example, the colored portion of the next diagram represents the graph of the nonstrict inequality $x \leq 4$:



4. Draw the graph of the inequality $x > -2$.

5. Draw the graph of the inequality $x \leq 0$.

6. What inequality goes with this graph?



7. How would you use inequalities to describe this graph?