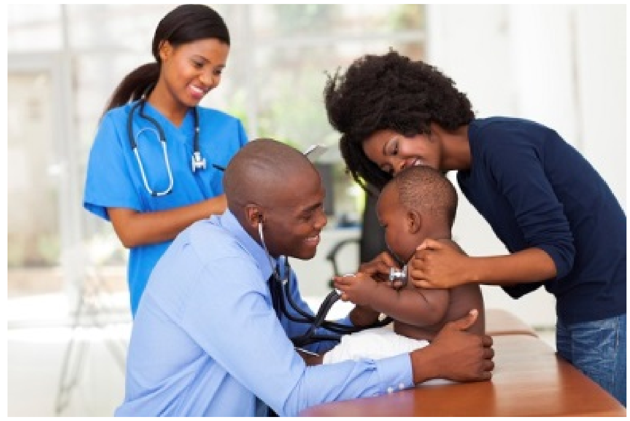
9.3: Reached Your Limit



Limit of a Sequence and Limits at infinity

When a patient receives medication, the size and frequency of the dose is very important. Some medicines cannot be given in large doses because the patient may react violently to the chemical change. Other medicines may be fatal if too much accumulates in the bloodstream, but would be of no value if too little is in the bloodstream. Sometimes the range of effective doses is very narrow. Each patient metabolizes medications differently; therefore, the rate of medication must be calculated for each patient. Doctors and pharmacists often use the concept of convergent sequences and limits to help determine medication schedules.

In order to understand the idea of the limit of a function $f(x)$ as $x \to \infty$, you will revisit convergent and divergent sequences. Recall that a sequence **converges** if it draws near to a value as n increases without bound. Also recall that if a sequence does not converge it is said to **diverge**. The *limit of a sequence* is the real number, L, that the sequence converges to and is denoted $\lim_{n\to\infty} a_n = L$ If a sequence diverges, the limit does not exist.

1. The n^{th} term of a sequence is described by $a_n = \frac{2}{n}$.
   a. Write the first 5 terms in the sequence.

   b. Complete the table below to find the indicated terms of the sequence

n	1	10	100	1000	10,000
a_n					

c. Use a graphing calculator to graph the sequence. Sketch the graph of the first 5 terms below.
Note: On the nSpire select Menu > 2 Graph Entry > 6 Sequence > 1 Sequence

d. Based on the evidence from the table and the graph, does the sequence converge or diverge?

e. What does $\lim_{n \to \infty} a_n$ appear to be? Explain your response.

2. The n^{th} term of a sequence is described by $a_n = \frac{2n-4}{n}$.

a. Write the first 5 terms in the sequence.

b. Complete the table below to find the indicated terms of the sequence

n	1	10	100	1000	10,000
a_n					

c. Use a graphing calculator to sketch the graph of the first 5 terms.

d. Based on the evidence from the table and the graph, does the sequence converge or diverge?

e. What does $\lim_{n \to \infty} a_n$ appear to be? Explain your response.

3. The n^{th} term of a sequence is described by $a_n = \frac{2n^2-4}{n}$.
   a. Write the first 5 terms in the sequence.

   b. Complete the table below to find the indicated terms of the Sequence

n	1	10	100	1000	10,000
a_n					

   c. Use a graphing calculator to sketch the graph of the first 5 terms.

   d. Based on the evidence from the table and the graph, does the sequence converge or diverge?

   e. What does $\lim_{n \to \infty} a_n$ appear to be? Explain your response.

4. Each of the sequences below is arithmetic. For each sequence, write an explicit formula for a_n and tell whether the sequence converges or diverges. If the sequence converges, give $\lim_{n \to \infty} a_n$.

   a. 7, 13, 19, 25, 31, …

   b. 2, -4, -10, -16, -22, …

   c. $\frac{1}{6}, 0, -\frac{1}{6}, -\frac{1}{3}, -\frac{1}{2}, \ldots$

   d. 4, 4, 4, 4, 4, …

5. Do all arithmetic sequences converge? Explain your reasoning.

6. Each of the sequences below is geometric. For each sequence, write an explicit formula for $a_n$and tell whether the sequence converges or diverges. If the sequence converges, give $\lim_{n \to \infty} a_n$.

   a. 2, 10, 50, 250, …

   b. -2, -2, -2, -2, …

   c. 3, -3, 3, -3, …

   d. 20, 5, $\frac{5}{4}$, $\frac{5}{16}$, …

7. Do all geometric sequences converge? Explain your reasoning.

II. Limits at Infinity

8. Sequences are functions with a domain such that $n \in N$. For example, $a_n = \frac{1}{n}$ can be written as $f(x) = \frac{1}{x}, x \in N$

   a. Compare and contrast the graphs of the sequence defined by $a_n = \frac{1}{n}$ and the function $f(x) = \frac{1}{x}, x \in \mathbb{R} - \{0\}$.

   b. Find $\lim_{n \to \infty} a_n$

   c. Find $\lim_{x \to \infty} f(x)$

9. How does finding the value to which a sequence converges help you find the *limit as x approaches infinity* of a function?

10. What impact does the divergence of a sequence have on the **limit as x approaches infinity** of the corresponding function?

11. In items 2 and 3 you found the limit as x approaches infinity of two sequences. Use your knowledge of those sequences to determine $\lim\limits_{n\to\infty} f(x)$ for each of the following functions.
    a. $f(x) = \frac{2x-4}{x}$
    b. $f(x) = \frac{2x^2-4}{x}$

12. Recall that a rational function is of the form $\frac{p(x)}{q(x)}$. Explain how to determine $\lim\limits_{n\to\infty} \frac{p(x)}{q(x)}$.

13. Evaluate the following limits. Show any calculations and sketch any graphs used.
    a. $\lim\limits_{x\to\infty} (6 - \frac{3}{x^2})$
    b. $\lim\limits_{x\to\infty} (2x - 5)$
    c. $\lim\limits_{x\to\infty} (x)$
    d. $\lim\limits_{x\to\infty} \left(\frac{6x^2}{x+2}\right)$
    e. $\lim\limits_{x\to\infty} 2(\frac{1}{2})^x$
    f. $\lim\limits_{x\to\infty} (\frac{3}{x^2+1})$