9.1 Parametric Equations

Practice Tasks

I. Concepts and Procedures

1. Determine whether or not (186, -268) is a point on the graph of $x = 6 + 2t,\ \ y = 5 - 3t$.

2. The function $y = f(x)$ is defined parametrically by $x = 2 - 3t,\ y = t^2 + 1$. Write y as a function of x and sketch the graph of the function f.

3. The function $y = f(x)$ is defined parametrically by $x = 1 + 4t,\ y = t^2 - 3$. Write y as a function of x and sketch the graph of the function f.

II. Problem Solving

1. A jungle and wildlife preserve extends 80 miles north and 120 miles east of the ranger station. The ranger leaves from a point 100 miles east of the station along the southern boundary to survey the area. He travels to a point 0.6 miles north and 0.5 miles west every minute. A lion leaves the west edge of the preserve 51 miles north of the station at the same time the ranger leaves the Southern border. Every minute the lion moves to a point 0.1 miles north and 0.3 miles east. The lion and the ranger are both traveling on linear paths.
   a. Sketch the paths of the lion and the ranger. Write the linear equations to represent their paths.
   b. Where do their paths cross?
   c. Find the speed in miles per minute that both the lion and the ranger are moving.
   d. Write parametric equations for both the lion and the ranger.
   e. How long does it take the lion and ranger to get to the point where their paths cross?
   f. What is the closest the lion and ranger get to each other to 3 decimal places?

g. Another lion is tracked and its movements can be described using the equations $x = 6 + 0.18T$ and $y = 53 + 0.06T$. Show that the lions are on the same path. Will they "meet up" before leaving the preserve? If so, when?

2. In a science fiction movie, a crack in the ground is to expand across a field, while Rhena, an actress, races to cross the field before it is cut in half by the crack. Let $t = 0$ correspond to the time the crack begins to expand and suppose it expands according to the parametric equations $x(t) = 6t$ and $y(t) = 25 + 2t$, where t is measured in seconds and distance is measured in feet. Rhena's path is given by $x(t) = 240 - 4.5t$ and $y(t) = 15 + 3t$.
   a. Find the point at which the paths of the crack and Rhena will cross.
   b. Find the amount of time it takes the crack to expand to the point of intersection and the time it takes Rhena to arrive at that point. Does Rhena get across the field before the crack splits it in half?

3. Because of your incredible math skills you have been offered an exciting job as a radar tracking operator in the UFO Division of the National Aeronautics Research Department. On your very first day a UFO suddenly appears on your screen 55 mm to the right and 5 mm up from the lower left hand corner. At that same time, you notice an airplane entering the screen 35 mm directly above the lower left hand corner of the screen. Each second the UFO's position has moved 2 mm left and 3 mm up and the airplane's position has moved 3 mm right and 1 mm up.
   a. Sketch the paths of the UFO and the airplane. Write the linear equations to represent their paths.
   b. Where do their paths cross?
   c. Find the speed the UFO and the airplane are moving in millimeters per second.
   d. Write parametric equations for both the UFO and the airplane.
   e. How long does it take the UFO and the airplane to get to the point where their paths cross?
   f. What is the closest the plane and the UFO get to each other? (3 decimal places)

4. Imagine you are piloting a small plane at an altitude of 12,000 feet preparing to land. Once you begin your descent to the runway, your altitude changes at a rate of -15 ft /sec. Your horizontal speed is 200 ft /sec. Write parametric equations to model the descent of your plane.

II. Reasoning

1. How are parametric equations similar to vectors for modeling the path of an object?