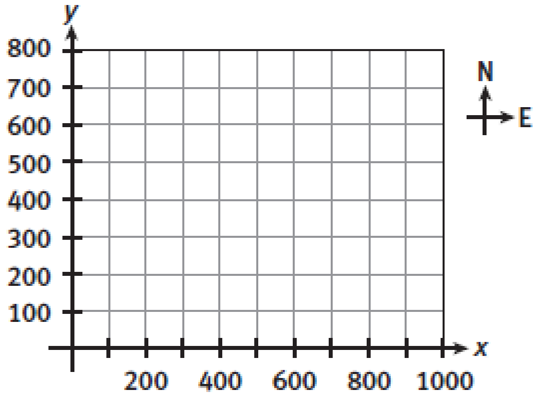
9.1: Ships in the Fog

Parametric Equations

Two ships are sailing in the fog. Both ships are being monitored by the same tracking equipment. As the ships enter the area being monitored, the *Andy Daria* is at a point 900 mm from the bottom left corner of the screen along the lower edge of the screen. The second ship, the *Helsinki,* is located 100 mm directly above the lower left hand corner.

One minute later the positions have changed. The *Andy Daria* has moved to a location on the screen that is 3 mm west and 2 mm north of its previous location. The *Helsinki* has moved to a position 4 mm east and 1 mm north of its previous position. Assume that the two ships will continue to sail at a constant speed on their respective paths.

1. Use the grid below, or GeoGebra, to graph and label points to represent the starting location for each fictional ship, *Andy Daria* (A) and *Helsinki* (H) and sketch the paths of each of the ships.



   a. Where will the two ships' paths cross?
   b. Will they collide at that point? If so, when?
   c. If not, how close do they actually come?

II. Parametric Equations

In the Opening Exercise, you investigated the paths of the two ships. Continue using the data below:

2. To find the coordinates of the point on the grid where the paths cross, you can write functions for the paths of the two ships.
    a. What is there about the ships' motion that tells you the paths were linear?
    b. What speeds are represented by the two ships on the tracking screen?
    c. Give the linear equation for each ship's path.

One question remains. How close did the ships actually come to each other? To answer this item, you can analyze the positions of the ships in terms of the time elapsed since they first appeared on the grid, t.

3. Fill in the tables below to indicate the positions of the ships relative to the lower left -hand corner of the screen at any given time.

Andy Doria

t (in minutes)	Horizontal x	Vertical y
0		
1		
2		
3		
10		
t		

Helsinki

t (in minutes)	Horizontal x	Vertical y
0		
1		
2		
3		
10		
t		

4. Write the rules for $x(t)$ and $y(t)$ for each ship.
    a. For the *Andy Doria*:
        i. $x(t) =$
        ii. $y(t) =$
    b. For the *Helsinki*:
        i. $x(t) =$
        ii. $y(t) =$

The rules written in Item 4 have time as the independent variable.

Each $x(t)$ and $y(t)$ pair determine the location (x, y), of the ship at time t. When an independent variable is used to define two (or more) dependent variables, the rules that relate the variables are known as **parametric equations**.

5. Show algebraically that the parametric equations for the *Andy Doria* are equivalent to the linear function that you found in Item 4c.

6. Explain what the coefficient of the variables and the constant mean in the function $x(t) = at + b, y(t) = ct + d$ and how they relate to the linear function $y(x)$.

7. Determine the grid location of both the *Andy Doria* and the *Helsinki* at time $t = 3$ min, and then find the distance between the ships at that time.

8. Determine the location of each ship one hour after they first appeared on the screen and find the distance between the ships at that time.

9. Develop a function $f(t)$ that gives the distance between the ships on the grid at any time t, since the ships entered the tracking screen.

10. Use the function created in Exercise 9 and your graphing calculator to answer the following:
    a. Find the minimum distance between the two ships on the tracking screen in millimeters.

    b. Find the time at which the two ships are closest to each other.

    c. At the time the ships are closest, what are the coordinates of each ship's image on the grid?
        i. *Andy Doria* (_____ , _____)
        ii. *Helsinki* (_____ , _____)

    d. At the time the ships are closest, is either ship at the crossing point of the ships' path?

11. Due to the fog, the visibility in the area the ships are sailing is 0.25 nautical mile. The scale of the observer's grid is 1 mm = 0.05 nautical mile.
    a. Determine whether it would have been possible for the two captains to see the other ship through the fog and thus know how close they had come. Explain the methods you can use to arrive at your answer.

    b. How fast was each ship traveling in nautical miles per hour?

III. More Ships… More Fog

On the night of July 25, 1956 the *Stockholm* actually crashed into the *Andrea Doria* in the fog. Suppose that the *Stockholm* was traveling on the same path and at the same rate as the fictional ship, the *Helsinki*, and that the *Andrea Doria* was traveling at the same rate, but on a parallel path and starting on the same horizontal line as the fictional ship, the *Andy Doria*.

12. At what point on the tracking grid would the pixel representing the *Andrea Doria* have first appeared in order to put the two ships on a collision course with the *Stockholm* under the given conditions? Show your work or write an explanation to support your answer.

The movement of two more ships, *HMS Lincoln* and *USS Park*, are monitored on a similar tracking screen. The position of each ship is measured as x mm horizontally and y mm vertically from the lower left -hand corner of the screen.

13. The linear path of the HMS Lincoln is given by $y = 2x + 50$. If $x = 3t$, express y as a function of t.

14. The parametric equations in Item 13 give the position of the *HMS Lincoln* at any time t minutes. On the same screen an observer notices that the path of the *USS Park* is described by the parametric equations $x(t) = \frac{5}{2}t + 10$ and $y(t) = 5t + 70$. Compare the paths of the two ships. The observer is concerned because she believes that the two ships are on the same path.
    a. Show work to confirm or contradict the observer's belief.
    b. The observer calculates that the ships will collide at $t = k$ minutes. Find k or refute the observer's claim.

15. The function $y = f(x)$ is defined parametrically by $x = t + 2$ and $y = t^2 + 3$. Write y as a function of x and sketch the graph of the function f.