# 8.7 Carbon Dating

*Practice Tasks*

## I. Problem Solving

1. A particular bank offers 6% interest per year compounded monthly. Timothy wishes to deposit $1,000.
   a. What is the interest rate per month?

   b. Write a formula for the amount $A$ Timothy will have after $n$ months.

   c. Write a formula for the number of months it will take Timothy to have $A$ dollars.

   d. Doubling-Time is the amount of time it takes for an investment to double. What is the doubling-time of Timothy's investment?

   e. In general, what is the doubling-time of an investment with an interest rate of $\frac{r}{12}$ per month?

2. A study done from 1950 through 2000 estimated that the world population increased on average by 1.77% each year. In 1950, the world population was 2519 million.
   a. Write a formula for the world population $t$ years after 1950. Use $p$ to represent world population.

   b. Write a formula for the number of years it will take to reach a population of $p$.

   c. Use your equation in part (b) to find when the model predicts that the world population will be 10 billion.

3. Consider the case of a bank offering $r$ (given as a decimal) interest per year compounded monthly, if you deposit $P$ dollars.
   a. What is the interest rate per month?

   b. Write a formula for the amount $A$ you will have after $n$ months.

   c. Write a formula for the number of months it will take to have $A$ dollars.

   d. What is the doubling-time of an investment earning 7% interest per year, compounded monthly? Round up to the next month.

4. A half-life is the amount of time it takes for a radioactive substance to decay by half. In general, we can use the equation $A = P\left(\frac{1}{2}\right)^t$ for the amount of the substance remaining after $t$ half-lives.
   a. What does $P$ represent in this context?

   b. If a half-life is 20 hours, rewrite the equation to give the amount after $h$ hours.

   c. Use the natural logarithm to express the original equation as having base $e$.

   d. The formula you wrote in part (c) is frequently referred to as the "Pert" formula, that is, $Pe^{rt}$. Analyze the value you have in place for $r$ in part (c). What do you notice? In general, what do you think $r$ represents?

   e. Jess claims that any exponential function can be written with base $e$. Is she correct? Explain why.

5. If caffeine reduces by about 10% per hour, how many hours $h$ does it take for the amount of caffeine in a body to reduce by half (round up to the next hour)?

6. Iodine-123 has a half-life of about 13 hours, emits gamma-radiation, and is readily absorbed by the thyroid. Because of these facts, it is regularly used in nuclear imaging.
   a. Write a formula that gives you the percent $p$ of iodine-123 left after $t$ half-lives.

   b. What is the decay rate per hour of iodine-123? Approximate to the nearest millionth.

   c. Use your result to part (b). How many hours $h$ would it take for you to have less than 1% of an initial dose of iodine-123 in your system? Round your answer to the nearest tenth of an hour.

7. An object heated to a temperature of $50° C$ is placed in a room with a constant temperature of $10° C$ to cool down. The object's temperature $T$ after $t$ minutes can be given by the function $T(t) = 10 + 40e^{-0.023105t}$.
   a. How long will it take for the object to cool down to $30° C$?

   b. Will it take longer for the object to cool from $50° C$ to $30° C$ or from $30° C$ to $10.1° C$?

   c. Will the object ever be $10° C$ if kept in this room?

   d. What is the domain of $T^{-1}$? What does this represent?

8. The percent of usage of the word "judgment" in books can be modeled with an exponential decay curve. Let $P$ be the percent as a function of $x$, and let $x$ be the number of years after 1900, then $P(x) = 0.0220465 \cdot e^{-0.0079941x}$.

   a. According to the model, in what year was the usage 0.1% of books?

   b. When will the usage of the word "judgment" drop below 0.001% of books? This model was made with data from 1950 to 2005. Do you believe your answer will be accurate? Explain.

   c. Find $P^{-1}$. What does the domain represent? What does the range represent?

## II. Reasoning

1. How can the graph of $g(x) = \log_4 x$ be obtained by a transformation of $f(x) = \ln x$?