8.6: Inverse Log?

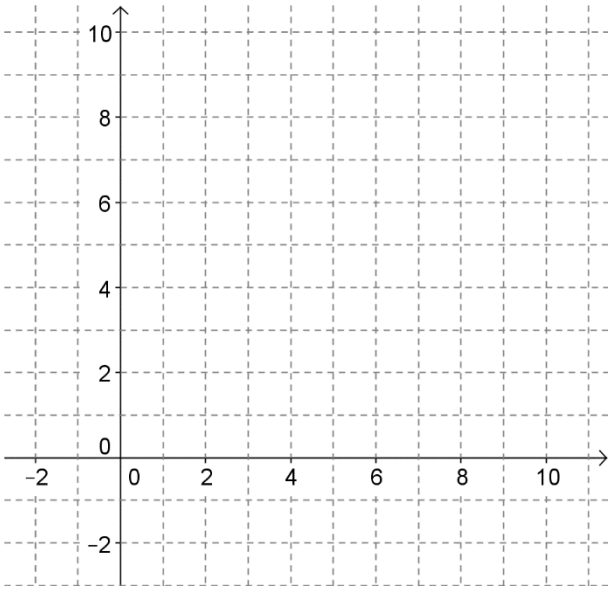
Log and Exponential Functions

So, do logarithmic functions have an inverse? Let's find out?

1. Let $f(x) = 2^x$.
   a. Complete the table

x	$f(x)$
-2	
-1	
0	
1	
2	
3	

   b. Use the points $(x, f(x))$ to create a sketch of the graph of $y = f(x)$.



c. Create a table of values for the function f^{-1}, and sketch the graph of $y = f^{-1}(x)$ on the grid above.

d. What type of function is f^{-1}? Explain how you know.

II. Getting Back on the Log

2. Given $f(x) = 2^x$, use the definition of the inverse of a function and the definition of a logarithm to write a formula for $f^{-1}(x)$.

3. Find the value of y in each equation. Explain how you determined the value of y.

   a. $y = \log_2(2^2)$

   b. $y = \log_2(2^5)$

   c. $y = \log_2(2^{-1})$

   d. $y = \log_2(2^x)$

4. Let $f(x) = \log_2(x)$ and $g(x) = 2^x$.
   a. What is $f\big(g(x)\big)$?

   b. Based on the results of part (a), what can you conclude about the functions f and g?

5. Find the value of y in each equation. Explain how you determined the value of y?
   a. $y = 3^{\log_3(3)}$

   b. $y = 3^{\log_3(9)}$

   c. $y = 3^{\log_3(81)}$

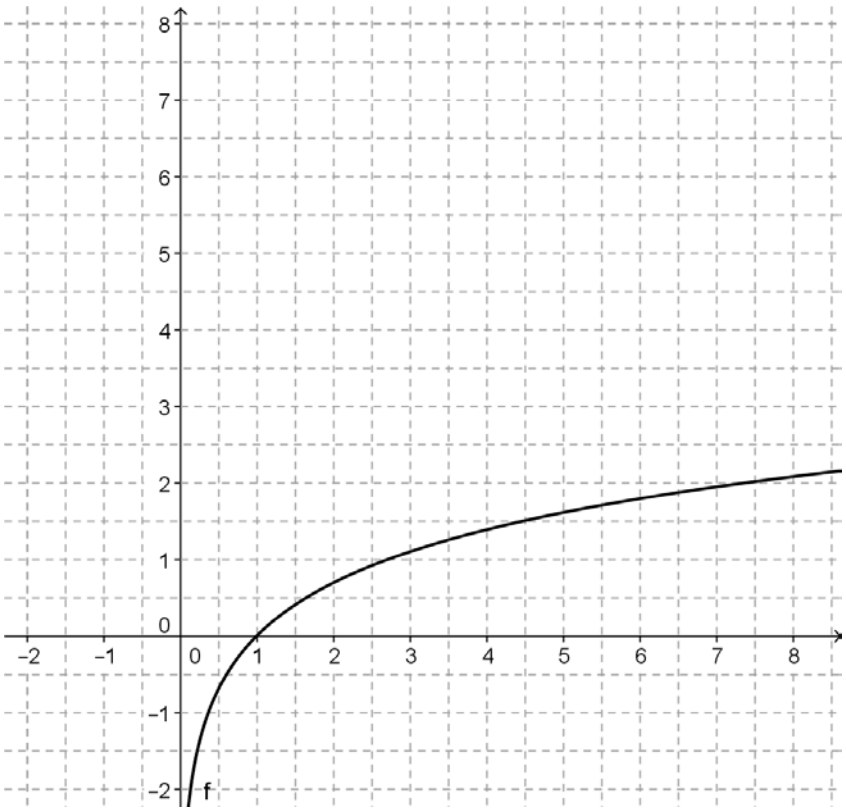
   d. $y = 3^{\log_3(x)}$

6. Let $f(x) = \log_3(x)$ and $g(x) = 3^x$.
   a. What is $g\big(f(x)\big)$?

   b. Based on the results in part (a), what can you conclude about the functions f and g?

7. Verify by composition that the functions $f(x) = b^x$ and $g(x) = \log_b(x)$ for $b > 0$ are inverses of one another.

8. The graph of $y = f(x)$, a logarithmic function, is shown below.



a. Construct the graph of $y = f^{-1}(x)$.

b. Estimate the base b of these functions. Explain how you got your answer.

III. It's Only Natural

9. Use a calculator to get a very accurate estimate of irrational number e.

10. Is the graph of $y = f^{-1}(x)$ in Exercise 6 a good approximation of the function $g(x) = e^x$? Explain your reasoning.

11. Show that $f(x) = \ln(x)$ and $g(x) = e^x$ are inverse functions by graphing $y = f(g(x))$ and $y = g(f(x))$ on a graphing calculator. Explain how your graphs support the fact that these two functions are indeed inverses of one another.

12. What is the base of the natural logarithm function $f(x) = \ln(x)$? Explain how you know.

13. Find the inverse of each function.

a. $f(x) = 2^{x-3}$

b. $g(x) = 2\log(x-1)$

c. $h(x) = \ln(x) - \ln(x-1)$

d. $k(x) = 5 - 3^{-\frac{x}{2}}$

14. Check your solutions to Exercise 13 by graphing the functions and the inverses that you found and verifying visually that the reflection property holds.