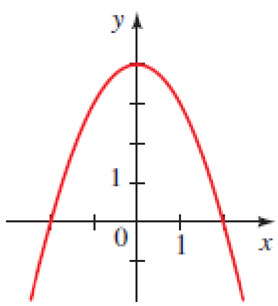
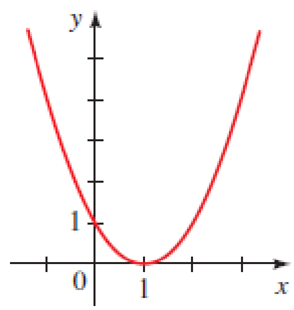
8.5 Restricting the Domain

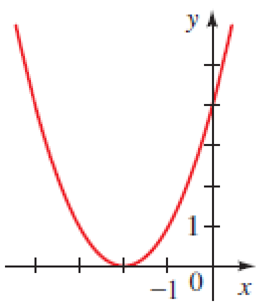
Practice Tasks

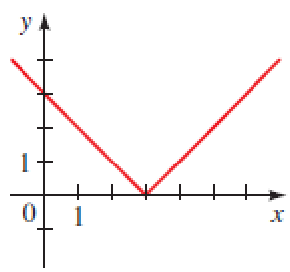
I. Concepts and Procedures

1. The inverses of the given functions are not functions. Restrict the domain so that the resulting inverse function *is* a function. Find the inverse of the function with the restricted domain. (There is more than one correct answer.)

a. 

b. 

c. 

d. 

2. Use function composition to prove that the following functions are inverses of each other. [Hint: $f\left(g(x)\right) = g\left(f(x)\right) = x$]

a. $f(x) = x - 6; \quad g(x) = x + 6$

b. $f(x) = 2x - 5; \quad g(x) = \frac{x+5}{2}$

c. $f(x) = x^5$; $g(x) = \sqrt[5]{x}$

d. $f(x) = \frac{1}{x-1}$, $x \neq 1$; $g(x) = \frac{1}{x} + 1$, $x \neq 0$

e. $f(x) = \frac{x+2}{x-2}$, $x \neq 2$; $g(x) = \frac{2x+2}{x-1}$, $x \neq 1$;

3. For each of the following, write the inverse of the function given.

a. $f(x) = 4 + \sqrt[3]{x}$

b. $f(x) = \frac{2x+5}{x-7}$

c. $\mathrm{f(x)} = (2 - x^3)^5$

II. Problem Solving

1. Marcello's Pizza charges a base price of \$7 for a large pizza plus \$2 for each topping. Thus, if you order a large pizza with x toppings, the price of your pizza is given by the function $f(x) = 7 + 2x$. Find f^{-1}. What does the function f^{-1} represent?

2. A car dealership advertises a 15% discount on all its new cars. In addition, the manufacturer offers a \$1000 rebate on the purchase of a new car. Let x represent the sticker price of the car.
   a. Suppose only the 15% discount applies. Find a function f that models the purchase price of the car as a function of the sticker price x.

   b. Suppose only the \$1000 rebate applies. Find a function g that models the purchase price of the car as a function of the sticker price x.

   c. Find a formula for $H = f \circ g$.

   d. Find H^{-1}. What does H^{-1} represent?

   e. Find $H^{-1}(13{,}000)$. What does your answer represent?

III. Reasoning

1. Let f be the function that assigns to each student in your class his or her biological mother.
   a. In order for f to have an inverse, what condition must be true about the students in your class?
   b. If we enlarged the domain to include all students in your school, would this larger domain function have an inverse? Explain.

2. Consider a linear function of the form $f(x) = mx + b$, where m and b are real numbers, and $m \neq 0$.
   a. Explain why linear functions of this form always have an inverse this is also a function.
   b. State the general form of a line that does not have an inverse.
   c. What kind of function is the inverse of an invertible linear function (e.g., linear, quadratic, exponential, logarithmic, rational, etc.)?
   d. Find the inverse of a linear function of the form $f(x) = mx + b$, where m and b are real numbers, and $m \neq 0$.

3. Consider a quadratic function of the form $f(x) = b\left(\frac{x-h}{a}\right)^2 + k$ for real numbers a, b, h, k, and $a, b \neq 0$.
   a. Explain why quadratic functions never have an inverse without restricting the domain.
   b. What are the coordinates of the vertex of the graph of f?
   c. State the possible domains you can restrict f on so that it will have an inverse.
   d. What kind of function is the inverse of a quadratic function on an appropriate domain?
   e. Find f^{-1} for each of the domains you gave in part (c).

4. Show that $f(x) = mx + b$ for real numbers m and b with $m \neq 0$ has an inverse that is also a function.

5. Explain why $f(x) = a(x-h)^2 + k$ for real numbers a, h, and k with $a \neq 0$ does not have an inverse that is a function. Support your answer in at least two different ways (numerically, algebraically, or graphically).

IV. Modeling

1. Consider the function $f(x) = \sin(x)$.
   a. Graph $y = f(x)$ on the domain $[-2\pi, 2\pi]$.
   b. If we require a restricted domain on f to be continuous and cover the entirety of the range of f, how many possible choices for a domain are there in your graph from part (a)? What are they?
   c. Make a decision on which restricted domain you listed in part (b) makes the most sense to choose. Explain your decision.
   d. Use a calculator to evaluate $\sin^{-1}(0.75)$ to three decimal places. How can you use your answer to find other values ψ such that $\sin(\psi) = 1$? Verify that your technique works by checking it against your graph in part (a).