## 8.5: Restrict Your Twitter!

Restricting the Domain to
Make a Function Invertible

You have 10 minutes to try and complete this challenge... or complete more than anyone in the
 class!

Can you create the Twitter Bird logo on Desmos or some other graphing utility? There are 12 algebraic expressions, each with a restricted domain.

- Save the image (a full-size version on the last page of this lesson)
- Add image (Plus sign on Desmos)
- Move the image so the bottom-left corner is at the origin
- Type in the function
- Add the restriction to the end of your equation in curly brackets $\}$
o Example: $y=.25(x-2)^{2}\{0<x<5\}$ will restrict a parabola to a domain of $0<x<5$


Thanks to Jennifer Silverman. You can submit your Desmos designs too!

## II. Restricting the Domain of a Function

You may recall that a function can be a simple mapping that assigns each element in the domain to a corresponding element in the range. The function $f$ shown below pairs each element in the domain set with one element in the range.

1. The function $f$ with domain $\{1,2,3,4,5\}$ is shown in the table below.

| $x$ | $f(x)$ |
| :---: | :---: |
| 1 | 7 |
| 2 | 3 |
| 3 | 1 |
| 4 | 9 |
| 5 | 5 |

a. What is $f(1)$ ? Explain how you know.
b. What is $f^{-1}(1)$ ? Explain how you know.
c. What is the domain of $f^{-1}$ ? Explain how you know.
d. Construct a table for the function $f^{-1}$, the inverse of $f$.
2. Complete the mapping diagram to show that $f\left(f^{-1}(x)\right)=x$.

3. Complete the mapping diagram to show that $f^{-1}(f(x))=x$.

4. The graph of $f$ is shown below.
a. Select several ordered pairs on the graph of $f$, and use those to construct a graph of $f^{-1}$.

b. Draw the line $y=x$, and use it to construct the graph of $f^{-1}$ below.

c. The algebraic function for $f$ is given by $f(x)=x^{3}+2$. Is the formula for $f^{-1}(x)=$ $\sqrt[3]{x}-2$ ? Explain why or why not.
5. The graph of $f(x)=\sqrt{x-3}$ is shown below. Construct the graph of $f^{-1}$.

6. Your classmate Morgan used the procedures learned in the previous lesson to define $f^{-1}(x)=x^{2}+3$. How does the graph of this function compare to the one you made in Exercise 5?
7. Construct the inverse of the function $f$ given by the table below. Is the inverse a function? Explain your reasoning.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 4 | -1 | -4 | -5 | -4 | -1 | 4 |

8. The graphs of several functions are shown below. Which ones are invertible? Explain your reasoning.






9. Given the function $f(x)=x^{2}-4$.
a. Select a suitable domain for $f$ that will make it an invertible function. State the range of $f$.
b. Write a formula for $f^{-1}$. State the domain and range of $f^{-1}$.
c. Verify graphically that $f$, with the domain you selected, and $f^{-1}$ are indeed inverses.
d. Verify that $f$ and $f^{-1}$ are indeed inverses by showing that $f\left(f^{-1}(x)\right)=x$ and $f^{-1}(f(x))=x$.
10. Three pairs of functions are given below. For which pairs, are $f$ and $g$ inverses of each other? Show work to support your reasoning. If a domain is not specified, assume it is the set of real numbers.
a. $\quad f(x)=\frac{x}{x+1}, x \neq-1$ and $g(x)=\frac{-x}{x-1}, x \neq 1$
b. $f(x)=\sqrt{x}-1, x \geq 0$ and $g(x)=(x+1)^{2}$
c. $\quad f(x)=-0.75 x+1$ and $g(x)=-\frac{4}{3} x-\frac{4}{3}$

## Lesson Summary

Composition of a Function and Its Inverse: To verify that two functions are inverses, show that $f(g(x))=x$ and $g(f(x))=x$.

Invertible Function: The domain of a function $f$ can be restricted to make it invertible.
A function is said to be invertible if its inverse is also a function.

Twitter Logo - For Opening Activity


