### 8.4 Inverse Functions

## Practice Tasks

## I. Concepts and Procedures

1. For each of the following, write the inverse of the function given.
a. $f=\{(1,3),(2,15),(3,8),(4,-2),(5,0)\}$
b. $\quad g=\{(0,5),(2,10),(4,15),(6,20)\}$
c. $h=\{(1,5),(2,25),(3,125),(4,625)\}$
d.

| $x$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 3 | 12 | 27 | 48 |

e.

| $x$ | -1 | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: |
| $g(x)$ | 3 | 6 | 12 | 24 |

f.

| $x$ | 1 | 10 | 100 | 1000 |
| :---: | :---: | :---: | :---: | :---: |
| $h(x)$ | 0 | 1 | 2 | 3 |

g. $y=2 x$
h. $y=\frac{1}{3} x$
i. $\quad y=x-3$
j. $\quad y=-\frac{2}{3} x+5$
k. $2 x-5 y=1$
l. $-3 x+7 y=14$
m. $y=\frac{1}{3}(x-9)^{3}$
n. $y=\frac{5}{3 x-4}, x \neq \frac{4}{3}$
o. $y=2 x^{7}+1$
p. $y=\sqrt[5]{x}$
q. $\quad y=\frac{x+1}{x-1}, x \neq 1$
2. For each part in Problem 1, state the domain, $D$, and range, $R$, of the inverse function.
3. Sketch the graph of the inverse function for each of the following functions.
a.

b.
C.

d.


## II. Problem Solving

1. Gavin purchases a new $\$ 2,995$ computer for his business, and when he does his taxes for the year, he is given the following information for deductions on his computer (this method is called MACRS—Modified Accelerated Cost Recovery System):

| Period | Calculation for Deduction | Present Value |
| :--- | :---: | :---: |
| First Year | $D_{1}=\frac{P_{0}}{5} \times 200 \% \times 50 \%$ | $P_{0}-D_{1}=P_{1}$ |
| Second <br> Year | $D_{2}=\frac{P_{1}}{5} \times 200 \%$ | $P_{1}-D_{2}=P_{2}$ |
| Third <br> Year | $D_{3}=\frac{P_{2}}{5} \times 200 \%$ | $P_{2}-D_{3}=P_{3}$ |

Where $P_{0}$ represents the value of the computer new.
a. Construct a table for the function $D$, giving the deduction Gavin can claim in year $x$ for his computer, $x=\{1,2,3\}$.
b. Find the inverse of $D$.
c. Construct a table for the function $P$, giving the present value of Gavin's computer in year $x, x=\{0,1,2,3\}$.
d. Find the inverse of $P$.
2. Problem 1 used the MACRS method to determine the possible deductions Gavin could have for the computer he purchased. The straight-line method can be used also. Assume the computer has a salvage value of $\$ 500$ after 5 years of use; call this value $S$. Then Gavin would be presented with this information when he does his taxes:

| Period | Calculation for Deduction | Present Value |
| :--- | :---: | :---: |
| First Year | $D_{1}=\left(P_{0}-S\right) / 5 \times 50 \%$ | $P_{0}-D_{1}=P_{1}$ |
| Second <br> Year | $D_{2}=\left(P_{0}-S\right) / 5$ | $P_{1}-D_{2}=P_{2}$ |
| Third <br> Year | $D_{3}=\left(P_{0}-S\right) / 5$ | $P_{2}-D_{3}=P_{3}$ |
| Fourth <br> Year | $D_{4}=\left(P_{0}-S\right) / 5$ | $P_{3}-D_{4}=P_{4}$ |
| Fifth Year | $D_{5}=\left(P_{0}-S\right) / 5$ | $S$ |

a. Construct a table for the function $D$, giving the deduction Gavin can claim in year $x$ for his computer in $x=\{1,2,3,4,5\}$.
b. What do you notice about the function for deduction in this problem compared to the function in Problem 9?
c. If you are given the deduction that Gavin claims in a particular year using the straight-line method, is it possible for you to know what year he claimed it in? Explain. What does this tell us about the inverse of $D$ ?

## III. Reasoning

1. Natalie thinks that the inverse of $f(x)=x-5$ is $g(x)=5-x$. To justify her answer, she calculates $f(5)=0$ and then finds $g(0)=5$, which gives back the original input.
a. What is wrong with Natalie's reasoning?
b. Show that Natalie is incorrect by using other examples from the domain and range of $f$.
c. Find $f^{-1}(x)$. Where do $f^{-1}(x)$ and $g(x)$ intersect?
2. Sketch a graph of the inverse of each function graphed below by reflecting the graph about the line $\boldsymbol{y}=\boldsymbol{x}$. State whether or not the inverse is a function.
a.

b.


3. How can you tell before you reflect a graph over $\boldsymbol{y}=\boldsymbol{x}$ if its reflection will be a function or not?
4. After finding several inverses, Callahan exclaims that every invertible linear function intersects its inverse at some point. What needs to be true about the linear functions that Callahan is working with for this to be true? What is true about linear functions that do not intersect their inverses?
5. If $f$ is an invertible function such that $f(x)>x$ for all $x$, then what do we know about the inverse of $f$ ?
