## 8.4: It's Easy As ABC

## Applying Inverse Functions

Businesses must track the value of their assets over time. When a business buys equipment, the value of the equipment is
 reduced over time. For example, electric companies provide trucks for their workers when they go out into the field to repair electrical lines. These trucks lose value over time but are still part of the business assets. For accounting purposes, many businesses use a technique called straight-line depreciation to calculate the value of equipment over time.

Suppose ABC Electric purchases a new work truck for $\$ 34,500$. They estimate that the truck's value will depreciate to $\$ 0$ over 15 years. The table below shows the value $v(t)$ of the truck in thousands of dollars depreciated over time $t$ in months using a straight-line depreciation method.

| $t$ | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v(t)$ | 34.5 | 32.2 | 29.9 | 27.6 | 25.3 | 23.0 | 20.7 | 18.4 | 16.1 |

1. Does the function $v$ appear to be a linear function? Explain your reasoning.
2. What is an appropriate domain and range for $v$ in this situation?
3. Write a formula for $v$ in terms of $t$, the months since the truck was purchased.
4. What will the truck be worth after 30 months? 40 months? 50 months?
5. When will the truck be valued at $\$ 30,000$ ? $\$ 20,000$ ? $\$ 10,000$ ?

## II. Inverse Functions

6. Construct a table that shows the time of depreciation, $t(v)$, in months as a function of the value of the truck, $v$, in thousands of dollars.
7. Does the function $t$ appear to be a linear function? Explain your reasoning.
8. What is an appropriate domain and range for $t$ in this situation?
9. Write a formula for $t$ in terms of the value of the truck, $v$, since it was purchased.
10. Explain how you can create the formula for $t$ using the formula for $v$ from Exercise 5 .

The functions $v$ and $t$ are what we call inverse functions. The definition is provided below:
The Inverse of a Function: Let $f$ be a function with domain set $X$ and range set $Y$. Then $f$ is invertible if there exists a function $g$ with domain $Y$ and range $X$ such that $f$ and $g$ satisfy the property:
For all $x$ in $X$ and $y$ in $Y, f(x)=y$ if and only if $g(y)=x$.
The function $g$ is called the inverse of $f$ and is often denoted $f^{-1}$.
If $f$ and $g$ are inverses of each other, then
The domain of $f$ is the same set as the range of $g$.
The range of $f$ is the same set as the domain of $g$.

## III. Symmetry in Inverse Functions

11. Sketch a graph of the equations $y=v(t)$ and $y=t(v)$ in the Cartesian plane. How do their graphs compare?
12. What is the meaning of the intersection point of the graphs of the two equations?
13. Add the graph of $y=x$ to your work in Exercise 11. Describe the relationship between the graphs of $y=v(\mathrm{t}), y=t(v)$, and $y=x$.

14. ABC Electric uses this formula, $f(x)=750-10 x$, to depreciate computers, where $f$ is the value of a computer and $x$ is the number of months since its purchase.
a. Calculate $f(36)$. What is the meaning of $f(36)$ ?
b. What is the meaning of $b$ in $f(b)=60$ ? What is the value of $b$ ?
c. Write a formula for $f^{-1}$, and explain what it means in this situation.
d. When will the depreciated value of a computer be less than $\$ 400$ ?
e. What is the meaning of $c$ in $f^{-1}(c)=60$ ? What is the value of $c$ ?

## IV. Finding the Inverse of a Function

To find the inverse of a function, all you have to do is solve for x and then switch the x and y terms!

EXAMPLE: Find the inverse of the function $f(x)=3 x-2$.

| $y=3 x-2$ | Write $y=f(x)$ |
| :---: | :--- |
| $3 x=y+2$ | Add 2 |
| $x=\frac{y+2}{3}$ | Divide by 3 |
| $f^{-1}(x)=\frac{x+2}{3}$ | Interchange $x$ and $y$ |

15. Find the inverses of the following functions.
a. $f(x)=\frac{2}{3} x-10$
b. $g(x)=2(x+4)^{3}$
c. $\quad h(x)=\frac{1}{x-2}, x \neq 2$
