

8.3: Rational Behavior

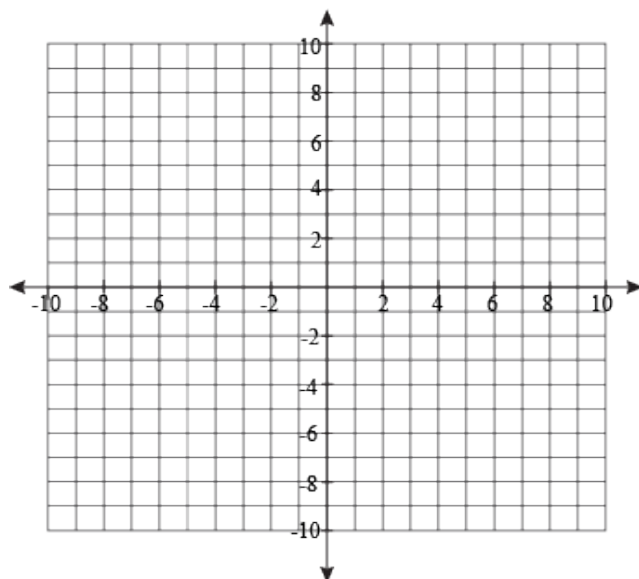
Analyzing Behavior of Rational Functions

The family of rational functions has a variety of characteristics. In this lesson you will become a Rational Master!



We will start by exploring the simplest rational function.

1. Graph the function $f(x) = \frac{1}{x}$ with a graphing device. Sketch the graph below.



- a. What are the domain and range of the function?
- b. Is the function continuous? If not, state the value(s) of x where there is a discontinuity in the graph.
- c. Why is there a discontinuity at that particular value?

The following tables show that when x is close to zero, the value of $|f(x)|$ is large, and the closer x gets to zero, the larger $|f(x)|$ gets.

x	$f(x)$	x	$f(x)$
-0.1	-10	0.1	10
-0.01	-100	0.01	100
-0.00001	-100,000	0.00001	100,000

Approaching 0^-

Approaching $-\infty$

Approaching 0^+

Approaching ∞

We describe this behavior in words and in symbols as follows. The first table shows that as x approaches 0 from the left side (the negative direction), the values of $y = f(x)$ decrease without bound.

In symbols, we write:

In words, we say:

$f(x) \rightarrow -\infty$ as $x \rightarrow 0^-$	"y approaches negative infinity as x approaches 0 from the left"
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The second table shows that as x approaches 0 from the right, the values of increase without bound. In symbols,

$f(x) \rightarrow \infty$ as $x \rightarrow 0^+$	"y approaches infinity as x approaches 0 from the right"
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The next two tables show how $|x|$ changes as $f(x)$ becomes large. (this is also known as the function's **end behavior**.)

x	$f(x)$	x	$f(x)$
-10	-0.1	10	0.1
-100	-0.01	100	0.01
-100,000	-0.00001	100,000	0.00001

Approaching $-\infty$

Approaching 0

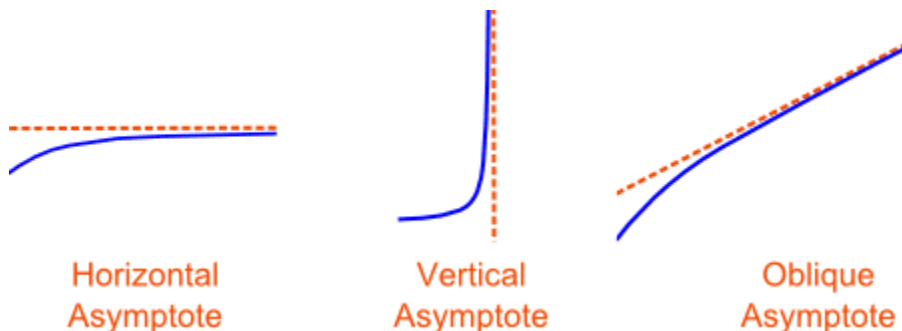
Approaching ∞

Approaching 0

2. Describe the end behavior of $f(x) = \frac{1}{x}$ by rewriting the following using symbols:
- The value of $f(x)$ gets closer and closer to zero, as x approaches negative infinity
 - The value of $f(x)$ gets closer and closer to zero, as x approaches infinity
3. Using appropriate symbols, analyze the end behavior of each function below.
- $f(x) = x^4$
 - $g(x) = -x^4$
 - $h(x) = x^3$
 - $k(x) = -x^3$
4. Determine the end behavior of each rational function below.
- $f(x) = \frac{x^2-3}{x^3}$
 - $f(x) = \frac{x^2-3}{x^2+1}$
 - $f(x) = \frac{7x^5-3x+1}{4x^3+2}$
 - $f(x) = \frac{7x^3-3x+1}{4x^3+2}$
 - $f(x) = \frac{7x^3+2}{4x^5-3x+1}$

II. Asymptotes

You may recall that the imaginary line that a graph seems to approach is called an **asymptote**. There are three types of asymptotes.



In analyzing rational behavior, you will be asked to identify any asymptotes. This is done by writing the equation of the line that forms the asymptote. For example, the graph of $f(x) = \frac{1}{x}$ that you sketched in Exercise 1 has a vertical asymptote of $x = 0$.

5. Consider the rational function $f(x) = \frac{2x-1}{x-4}$.

- a. State the domain of f .
- b. Determine the end behavior of f .
- c. State the horizontal asymptote of the graph of $y = f(x)$.
- d. Graph the function with a graphing calculator and make a sketch on your paper.

6. State the domain and end behavior of each rational function. Identify all horizontal and vertical asymptotes on the graph of each rational function. Then, verify your answer by graphing the function on the graphing calculator.

a. $f(x) = \frac{-x+6}{2x+3}$

b. $f(x) = \frac{3x-6}{x}$

c. $f(x) = \frac{3}{x^2-25}$

d. $f(x) = \frac{x^2-2}{x^2+2x-3}$

e. $f(x) = \frac{x^2-5x-4}{x+1}$

f. $f(x) = \frac{5x}{x^2+9}$

7. Write an equation for a rational function whose graph has the given characteristic. Graph your function on the graphing calculator to verify.

a. A horizontal asymptote of $y = 2$ and a vertical asymptote of $x = -2$.

b. A vertical asymptote of $x = 6$ and no horizontal asymptote.

c. A horizontal asymptote of $y = 6$ and no vertical asymptote.

8. State the domain of each of the following functions. Then, determine whether or not the excluded value(s) of x are vertical asymptotes on the graph of the function. Give a reason for your answer. Then check your answer by using a graphing calculator.

a. $f(x) = \frac{x^2 - 3x + 2}{x - 2}$

b. $f(x) = \frac{x^2 + 3x + 2}{x - 2}$

9. Sketch the graph of the rational function $f(x) = \frac{2x^2 - x}{x^2 - 16}$ showing all the key features of the graph. Label the key features on your graph.

10. Graph the function $f(x) = \frac{x^2 + 5x - 6}{x + 1}$ showing all key features.