8.1 Rational Operations

Practice Tasks



I. Concepts and Procedures

- 1. Given $\frac{x+1}{x-2}$ and $\frac{x-1}{x^2-4}$ show that performing the following operations results in another rational expression.
 - a. Addition.
 - b. Subtraction.
 - c. Multiplication.
 - d. Division.
- 2. For each pair of functions *f* and *g*, find the domain of *f* and the domain of *g*. Indicate whether *f* and *g* are the same function.

a.
$$f(x) = \frac{x^2}{x}, g(x) = x$$

b.
$$f(x) = \frac{2x^2 + 6x + 8}{2}$$
, $g(x) = x^2 + 6x + 8$

c.
$$f(x) = \frac{x^2 + 3x + 2}{x + 2}, g(x) = x + 1$$

d.
$$f(x) = \frac{x+2}{x^2+3x+2}$$
, $g(x) = \frac{1}{x+1}$

e.
$$f(x) = \frac{x^4 - 1}{x^2 - 1}, g(x) = x^2 + 1$$

f. $f(x) = \frac{x^4 - 1}{x^2 + 1}, g(x) = x^2 - 1$

3. Reduce each rational expression to lowest terms (i.e. simplified form), and specify any excluded values of *x*.

a.
$$f(x) = \frac{x^4}{x^2}$$

b.
$$f(x) = \frac{3x+3}{15x-6}$$

c.
$$f(x) = \frac{x^2 - x - 2}{x^2 + x}$$

d.
$$f(x) = \frac{8x^2 + 2x - 15}{4x^2 - 4x - 15}$$

e.
$$f(x) = \frac{2x^3 - 3x^2 - 2x + 3}{x^3 - x}$$

f.
$$f(x) = \frac{3x^3 + x^2 + 3x + 1}{x^3 + x}$$

II. **Problem Solving**

1. Find *A* and *B* that make the equation true. Verify your results.

a.
$$\frac{A}{x+1} + \frac{B}{x-1} = \frac{2}{x^2-1}$$
.

b.
$$\frac{A}{x+3} + \frac{B}{x+2} = \frac{2x-1}{x^2+5x+6}$$
.

2. Find *A*, *B*, and *C* that make the equation true. Verify your result.

$$\frac{Ax+B}{x^2+1} + \frac{C}{x+2} = \frac{x-1}{(x^2+1)(x+2)}.$$

- 3. Find two rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ that produce the result $\frac{x-1}{x^2}$ when using the following operations. Answers for each type of operation may vary. Justify your answers.
 - a. Addition.
 - b. Subtraction.
 - c. Multiplication.
 - d. Division.
- 4. Find two rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ that produce the result $\frac{2x+2}{x^2-x}$ when using the following operations. Answers for each type of operation may vary. Justify your answers.
 - a. Addition.
 - b. Subtraction.
 - c. Multiplication.
 - d. Division.

III. Reasoning

- 1. Consider the rational expressions *A*, *B* and their quotient, $\frac{A}{B}$, where *B* is not equal to zero.
 - a. For some rational expression *C*, does $\frac{AC}{BC} = \frac{A}{B}$?
 - b. Let $A = \frac{x}{y} + \frac{1}{x}$ and $B = \frac{y}{x} + \frac{1}{y}$. What is the least common denominator of every term of each expression?
 - c. Find *AC*, *BC* where *C* is equal to your result in part (b). Then find $\frac{AC}{BC}$. Simplify your answer.
 - d. Express each rational expression *A*, *B* as a single rational term; that is, as a division between two polynomials.
 - e. Write $\frac{A}{B}$ as a multiplication problem.
 - f. Use your answers to parts (d) and (e) to simplify $\frac{A}{R}$.
 - g. Summarize your findings. Which method do you prefer using to simplify rational expressions?