### 8.1 Rational Operations

## Practice Tasks

## I. Concepts and Procedures



1. Given $\frac{x+1}{x-2}$ and $\frac{x-1}{x^{2}-4}$ show that performing the following operations results in another rational expression.
a. Addition.
b. Subtraction.
c. Multiplication.
d. Division.
2. For each pair of functions $f$ and $g$, find the domain of $f$ and the domain of $g$. Indicate whether $f$ and $g$ are the same function.
a. $f(x)=\frac{x^{2}}{x}, g(x)=x$
b. $\quad f(x)=\frac{2 x^{2}+6 x+8}{2}, g(x)=x^{2}+6 x+8$
c. $\quad f(x)=\frac{x^{2}+3 x+2}{x+2}, g(x)=x+1$
d. $\quad f(x)=\frac{x+2}{x^{2}+3 x+2}, \quad g(x)=\frac{1}{x+1}$
e. $f(x)=\frac{x^{4}-1}{x^{2}-1}, g(x)=x^{2}+1$
f. $\quad f(x)=\frac{x^{4}-1}{x^{2}+1}, g(x)=x^{2}-1$
3. Reduce each rational expression to lowest terms (i.e. simplified form), and specify any excluded values of $x$.
a. $\quad f(x)=\frac{x^{4}}{x^{2}}$
b. $\quad f(x)=\frac{3 x+3}{15 x-6}$
c. $\quad f(x)=\frac{x^{2}-x-2}{x^{2}+x}$
d. $f(x)=\frac{8 x^{2}+2 x-15}{4 x^{2}-4 x-15}$
e. $f(x)=\frac{2 x^{3}-3 x^{2}-2 x+3}{x^{3}-x}$
f. $\quad f(x)=\frac{3 x^{3}+x^{2}+3 x+1}{x^{3}+x}$

## II. Problem Solving

1. Find $A$ and $B$ that make the equation true. Verify your results.
a. $\frac{A}{x+1}+\frac{B}{x-1}=\frac{2}{x^{2}-1}$.
b. $\frac{A}{x+3}+\frac{B}{x+2}=\frac{2 x-1}{x^{2}+5 x+6}$.
2. Find $A, B$, and $C$ that make the equation true. Verify your result.

$$
\frac{A x+B}{x^{2}+1}+\frac{C}{x+2}=\frac{x-1}{\left(x^{2}+1\right)(x+2)} .
$$

3. Find two rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ that produce the result $\frac{x-1}{x^{2}}$ when using the following operations. Answers for each type of operation may vary. Justify your answers.
a. Addition.
b. Subtraction.
c. Multiplication.
d. Division.
4. Find two rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ that produce the result $\frac{2 x+2}{x^{2}-x}$ when using the following operations. Answers for each type of operation may vary. Justify your answers.
a. Addition.
b. Subtraction.
c. Multiplication.
d. Division.

## III. Reasoning

1. Consider the rational expressions $A, B$ and their quotient, $\frac{A}{B}$, where $B$ is not equal to zero.
a. For some rational expression $C$, does $\frac{A C}{B C}=\frac{A}{B}$ ?
b. Let $A=\frac{x}{y}+\frac{1}{x}$ and $B=\frac{y}{x}+\frac{1}{y}$. What is the least common denominator of every term of each expression?
c. Find $A C, B C$ where $C$ is equal to your result in part (b). Then find $\frac{A C}{B C}$. Simplify your answer.
d. Express each rational expression $A, B$ as a single rational term; that is, as a division between two polynomials.
e. Write $\frac{A}{B}$ as a multiplication problem.
f. Use your answers to parts (d) and (e) to simplify $\frac{A}{B}$.
g. Summarize your findings. Which method do you prefer using to simplify rational expressions?
