7.9 Composite Functions

Practice Tasks

I. Concepts and Procedures

1. Let $f(x) = x^2 - x,\ g(x) = 1 - x$. Find
   a. $f \circ g$
   b. $g \circ f$
   c. $g \circ g$
   d. $f \circ f$
   e. $f(g(2))$
   f. $g(f(-1))$

2. Let $f(x) = x^2,\ g(x) = x + 3$. Find
   a. $g\big(f(5)\big)$
   b. $f\big(g(5)\big)$
   c. $f\big(g(x)\big)$
   d. $g\big(f(x)\big)$
   e. $g\left(f\left(\sqrt{x+3}\right)\right)$

3. Let $f(x) = x^3,\ g(x) = \sqrt[3]{x}$. Find
   a. $f \circ g$
   b. $g \circ f$
   c. $f\big(g(8)\big)$
   d. $g\big(f(2)\big)$
   e. $f\big(g(-8)\big)$
   f. $g\big(f(-2)\big)$

4. Find the domain and range of the following functions:

   a. $f(x) = -x^2 + 2$

   b. $f(x) = \sqrt{4 - x}$

   c. $f(x) = |x|$

5. For the following, find $f\big(g(x)\big)$ and $g\big(f(x)\big)$, and state the domain.

   a. $f(x) = x^2 - x,\ \ g(x) = x - 1$

   b. $f(x) = x^2 - x,\ \ g(x) = \sqrt{x - 2}$

II. Problem Solving

1. A company has developed a new highly efficient solar panel. Each panel can produce 0.75 mw of electricity each day. According to the Los Angeles power authority, all the traffic lights in the city draw 0.5 mw of power per day.

   a. Write a function that represents the relationship between the number of solar panels installed and the amount of energy generated per day (in mwh). Define the input and output.

   b. Write a function that represents the relationship between the number of days and the energy in mw consumed by the traffic lights. (How many days can one mw provide?)

   c. Write a function that could be used to determine the number of days that the traffic lights will stay on based on the number of panels installed.

   d. Determine an appropriate domain and range for part (c).

   e. How many days can 20 panels power all the lights?

2. A water delivery person is trying to determine the relationship between the mass of the cylindrical containers he delivers and their diameter in centimeters. The density of the bottles is 1 g/cm^3. The height of each bottle is approximately 60 centimeters.
   a. Write a function that represents the relationship between the volume of the cylinder and its diameter.

   b. Write a function that represents the relationship between the mass and volume of the cylinder.

   c. Write a function that could be used to determine the mass of one cylinder based on its diameter. Interpret the equation in context.

   d. Determine an appropriate domain and range for part (c).

   e. What is the approximate mass of a cylinder with a diameter of 30 centimeters?

3. A gold mining company is mining gold in northern California. Each mining cart carries an average 500 kg of dirt and rocks that contain gold from the tunnel. For each 2 metric tons of material (dirt and rocks), the company can extract an average of 10 grams of gold. The average wholesale gold price is $20 per a gram.
   a. Write a function that represents the relationship between the mass of the material mined in metric tons and the number of carts. Define the input and output.

   b. Write a function that represents the relationship between the amount of gold and the materials. Define the input and output.

   c. Write a function that could be used to determine the mass of gold in metric tons as a function of the number of carts coming out from the mine.

   d. Determine an appropriate domain and range for part (c).

   e. Write a function that could be used to determine the amount of money the gold is worth in dollars and the amount of gold extracted in metric tons.

   f. How much gold can 40,000 carts of material produce?

   g. How much, in dollars, can 40,000 carts of material produce?

4. Bob operates hot air balloon rides for tourist at the beach. The hot air balloon rises, on average, at 100 feet per minute. At sea level, the atmospheric pressure is 29.9 inches Hg. Using a barometric meter, Bob notices that the pressure decreases by 0.5 inches Hg for each 500 feet the balloon rises.

   a. Write a function that represents the relationship between the height of the air balloon and the time spent to reach that height.

   b. Write a function that represents the relationship between the height of the air balloon and the atmospheric pressure being applied to the balloon.

   c. Write a function that could be used to determine the pressure on the hot air balloon based on the time it spends rising.

   d. Determine an appropriate domain and range for part (c).

   e. What is the reading on the barometer 10 minutes after the hot air balloon has left the ground?