7.7H High Noon and Sunset Shadows Combined

A Develop Understanding Task

In the task *High Noon and Sunset Shadows* we described the two-dimensional circular motion of a rider on a Ferris wheel by separating that motion into two components—the horizontal motion of the "high noon shadow" of the rider as it moved along the ground, and the vertical motion of the "sunset shadow" as it moved up and down along the wall of a building. Mathematicians refer to this process as *resolving the motion into its horizontal and vertical components.*

The following data was captured by filming a person's hand as she slowly traced an image in the air with the tip of a pencil. The first table captures the horizontal movement of the pencil—similar to watching the "high noon shadow" of the pencil moving across the floor. The second table captures the vertical movement of the pencil—similar to watching the "sunset shadow" of the pencil moving up or down the wall.

Table 1	
time (sec)	horizontal position (inches)
0	2
1	4
2	6
3	4
4	6
5	9
6	11
7	12
8	10
9	8
10	11
11	12
12	10
12	8
14	4

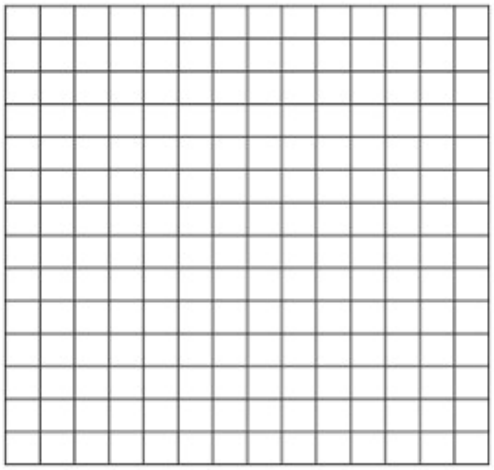
Table 2	
time (sec)	vertical position (inches)
0	11
1	9
2	12
3	3
4	12
5	13
6	12
7	10
8	7
9	8
10	7
11	4
12	2
12	2
14	3

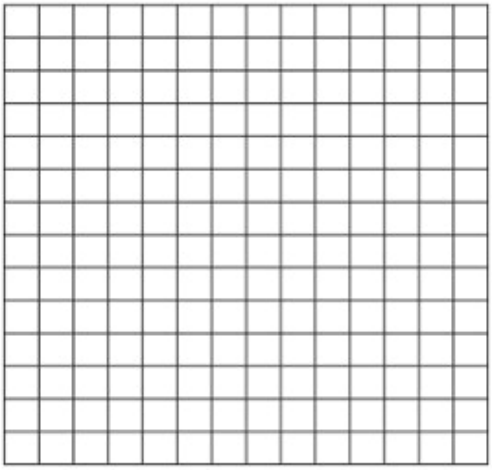
1. Examine table 1 and describe what it tells you about the horizontal motion of the person's hand.

2. Examine table 2 and describe what it tells you about the vertical motion of the person's hand.

3. The person is tracing a familiar letter in the air. Can you guess what the letter is? Explain how you made your guess.

4. At each second we know the horizontal and vertical location of the tip of the pencil. Plot points on the grid at the right to indicate these locations. Connect these points in a way that would show the location of the pencil at instances in time between the seconds given. Based on this graph, what letter do you think the person was tracing?





5. Draw a two-dimensional figure on the grid at the left. Create tables, like the ones above, to indicate where your pencil was at different moments in time as you drew your figure. Trade your tables with a partner and see if you can each replicate the figure that the other person drew based on the data you received.

Let's return to the rider on the Ferris wheel. The horizontal and vertical positions of the rider as a function of time are given by the following equations:

$$x(t) = 25\cos\left(\tfrac{\pi}{10}t\right)$$
$$y(t) = 25\sin\left(\tfrac{\pi}{10}t\right) + 30$$

6. How can you use these equations to determine the location of the rider at any instant in time? For example, how might you complete the following table?

time (seconds)	position of the rider
0	
1	
2	
3	
4	
5	
7.5	
10	
12.5	
15	
17.5	
20	

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Ready, Set, Go!

Ready

Topic: Equations of lines

Write the equation of the line (in slope-intercept form) that is defined by the given information.

1. $A(5,9)B(7,17)$

2. $P(-3,8)Q(-4,13)$

3. $G(3,-10)H(1,-11)$

4. $L(-5,6)M(-8,8)$

5.

x	y
1	-1
6	1
11	3

6.

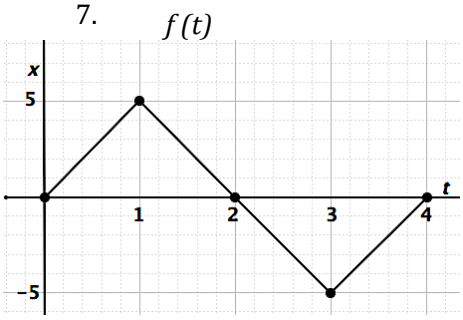
x	y
1	2
5	5
9	8

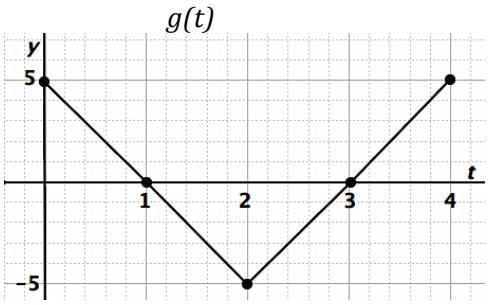
Set

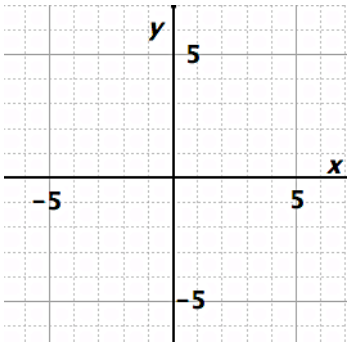
Topic: Parametric equations

The two given graphs show the motion of a particle whose position at time *t seconds* is given by $x = f(t)$ and $y = g(t)$. Describe the motion of the particle. Then graph the two graphs as one graph in the xy plane. Connect the points to indicate the motion at each second.

7.







Describe the motion:

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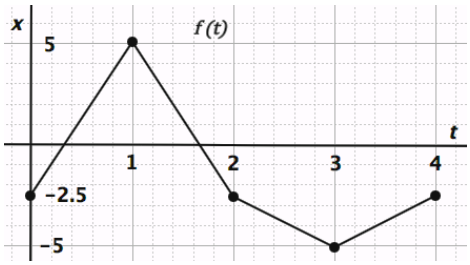
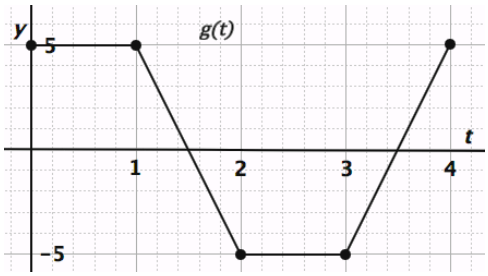
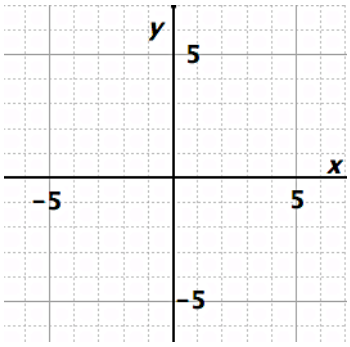
8.

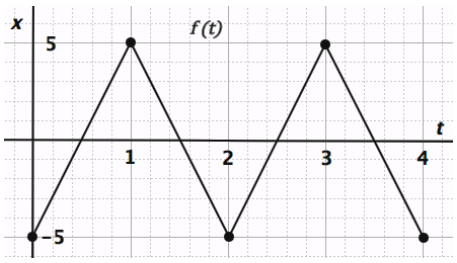


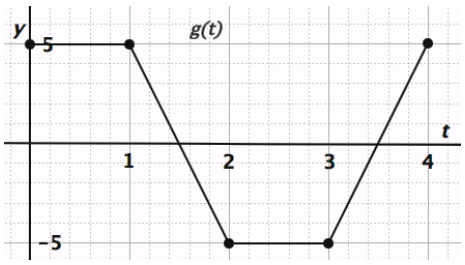


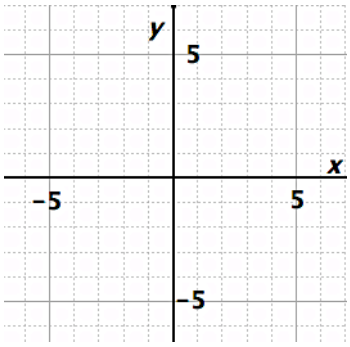


Describe the motion:

9.



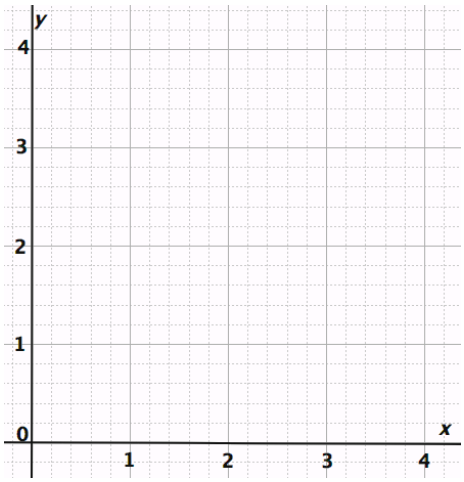




Describe the motion:

10. Fill in the table of values for the pair of parametric equations: $x = 3 + \sin t$ *and* $y = 2 + \cos t$. Sketch the graph of the two equations in the *xy* plane. Indicate the direction of the curve. Use the graph to write an equation for y as a function of x.

time (t)	$x = 3 + \sin t$	$y = 2 + \cos t$	(x, y)
0			
1			
2			
3			
4			
5			
6			
7			



equation:

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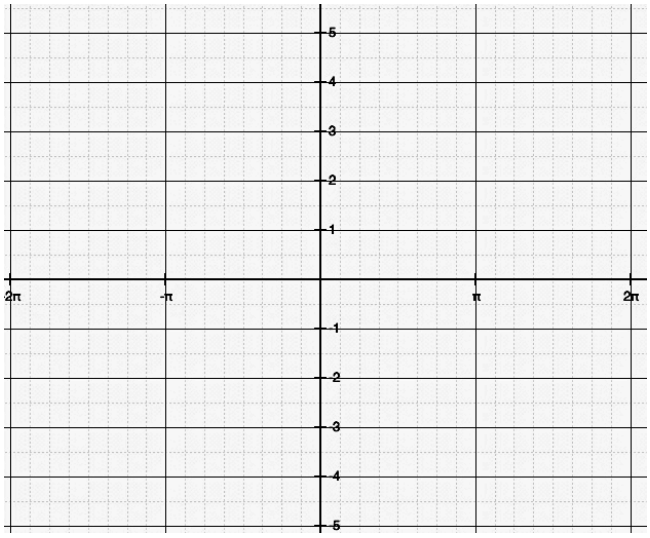
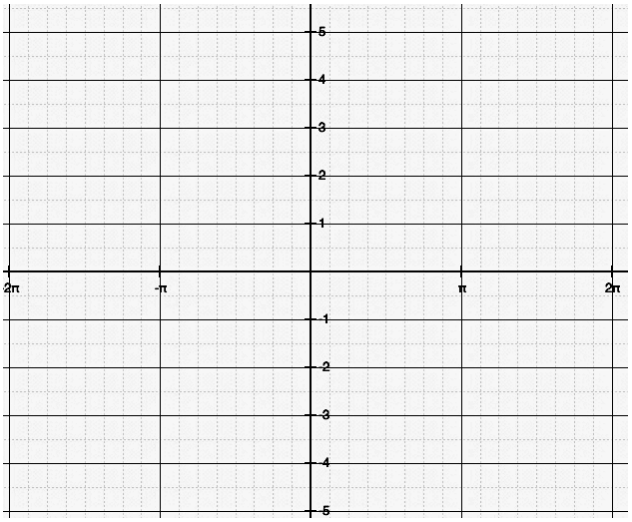
Go Topic: Graphs of the trigonometric functions

Graph the functions. $(-2\pi \leq \theta \leq 2\pi)$

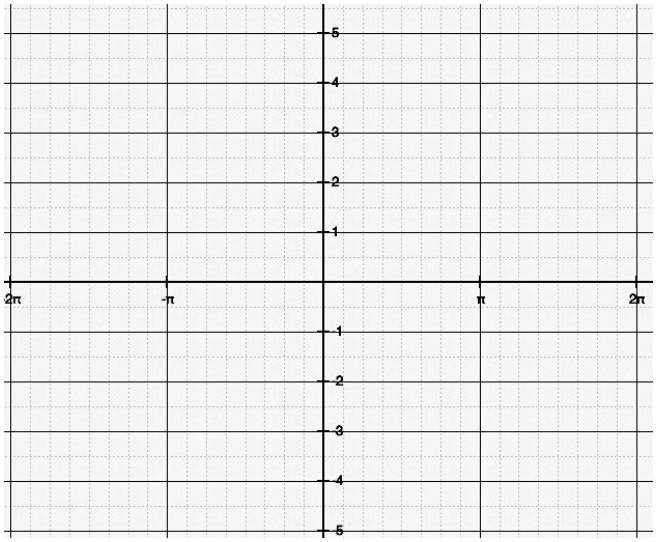
11. $y = 4 + \sin\theta$



12. $y = \tan\theta$



13. $y = -3 + 2\cos\theta$



14. $y = -1 - 3\cos\theta$